Computer Vision – Lecture 2

Binary Image Analysis

26.10.2016

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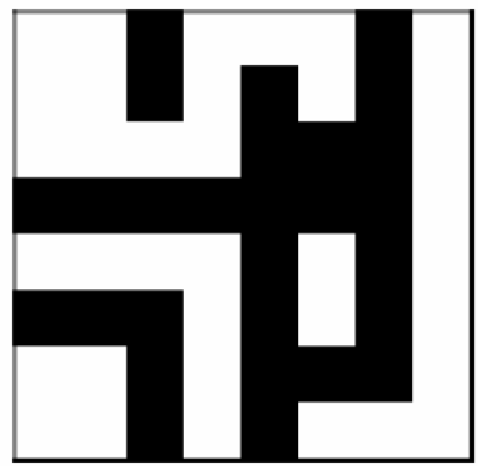
Announcements

- Course webpage
  - http://www.vision.rwth-aachen.de/courses/
  - Slides will be made available on the webpage
- L2P electronic repository
  - Exercises and supplementary materials will be posted on the L2P
- Please subscribe to the lecture on the Campus system!
  - Important to get email announcements and L2P access!
  - Bachelor students please also subscribe

Binary Images

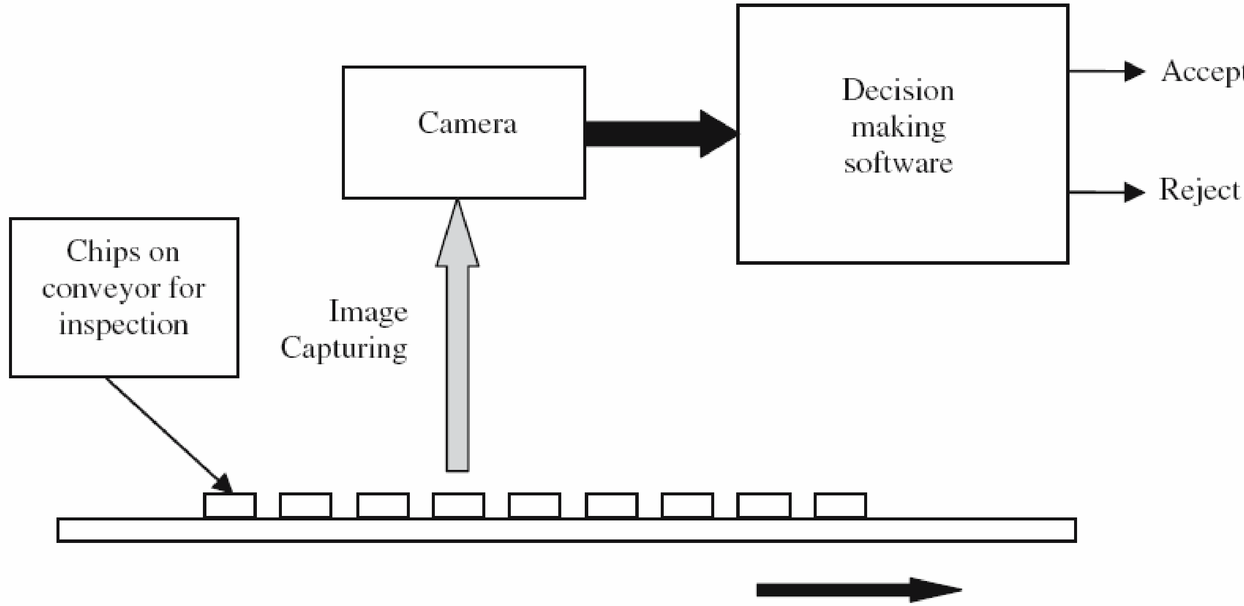
- Just two pixel values
- Foreground and background
- Regions of interest

1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	0	1	0	0	0	1
1	1	0	1	0	1	1	1



Uses: Industrial Inspection

Fig. 3 Schematic diagram of marking inspection setup at Texas Instruments



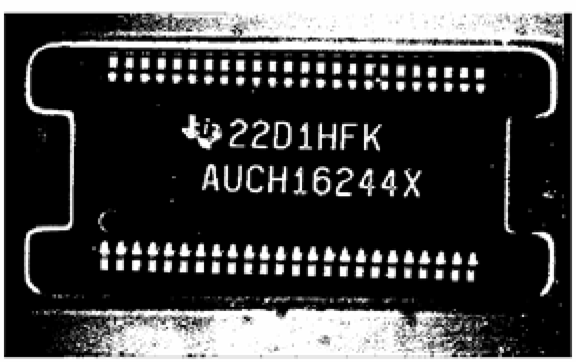
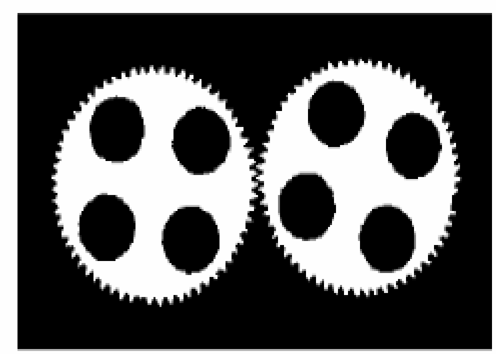


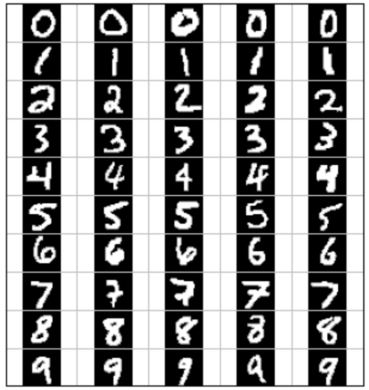
Fig. 7 Binarized image

Fig. 9 Row sum for separating a row



R. Nagarajan et al. “A real time marking inspection scheme for semiconductor industries“, 2006

Uses: Document Analysis, Text Recognition



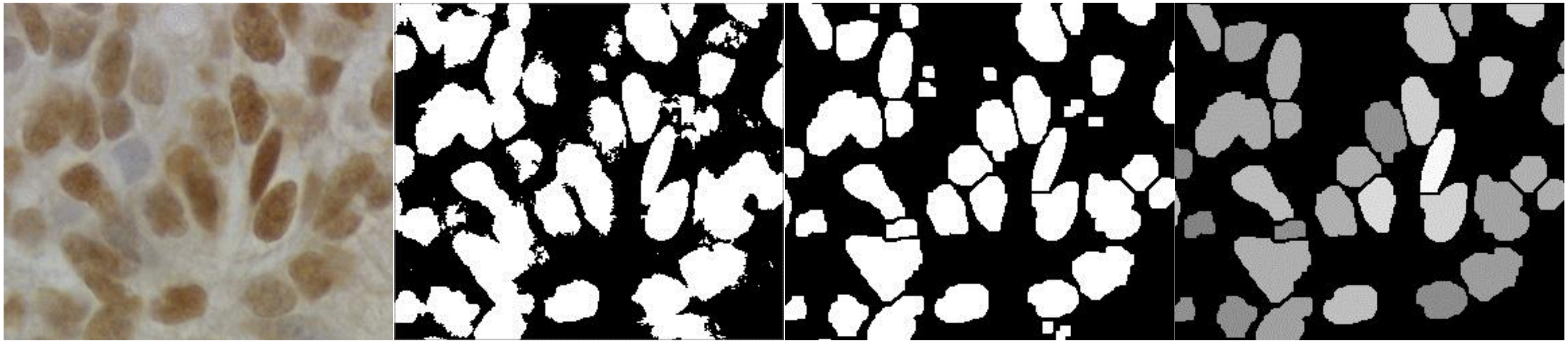
Handwritten digits

Scanned documents

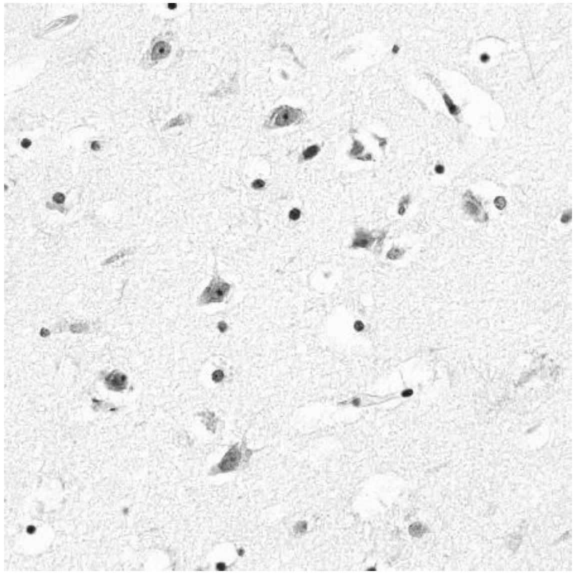
Natural text (after detection)

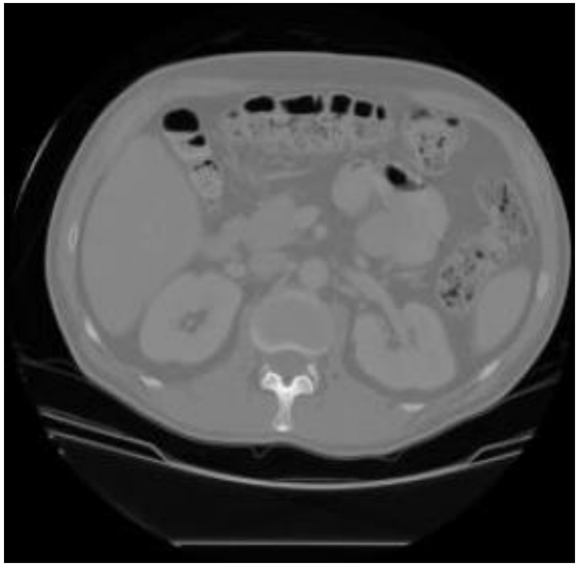
Source: Till Quack, Martin Renold

Uses: Medical/Bio Data



Source: D. Kim et al., Cytometry 35(1), 1999





Uses: Blob Tracking & Motion Analysis

Frame Differencing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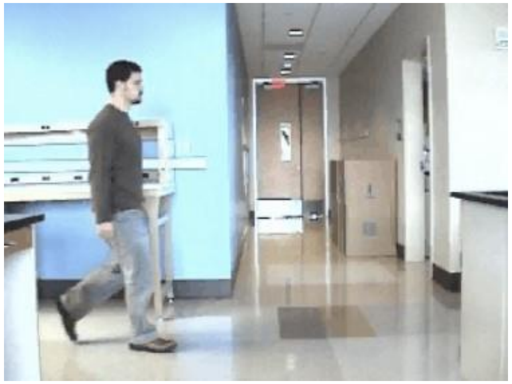
Source: Kristen Grauman

Background Subtraction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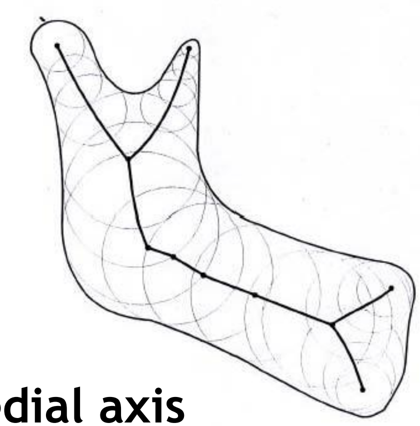
Source: Tobias Jäggli

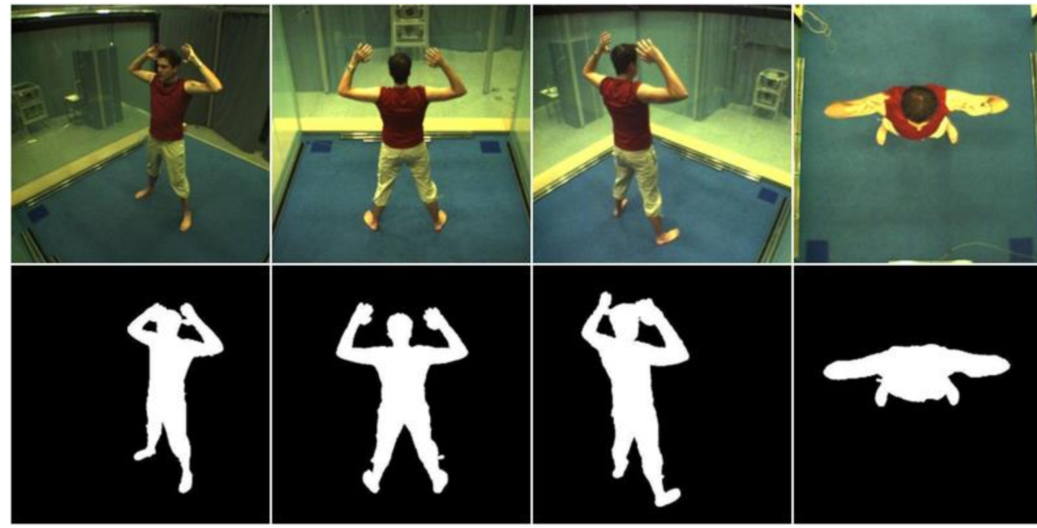
Uses: Shape Analysis, Free-Viewpoint Video

Silhouette

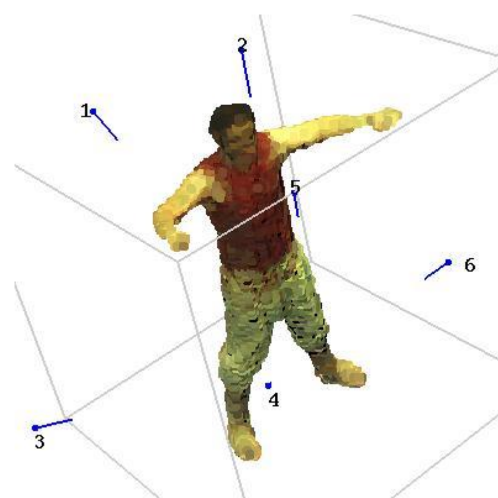


Medial axis

Visual Hull Reconstruction

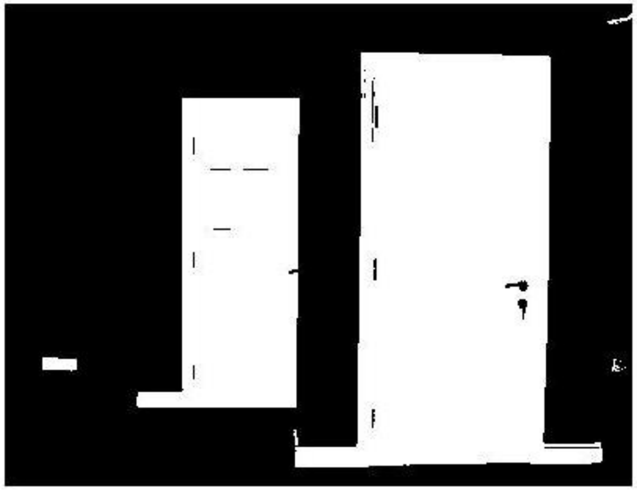


Blue-c project, ETH Zurich



Uses: Intensity Based Detection

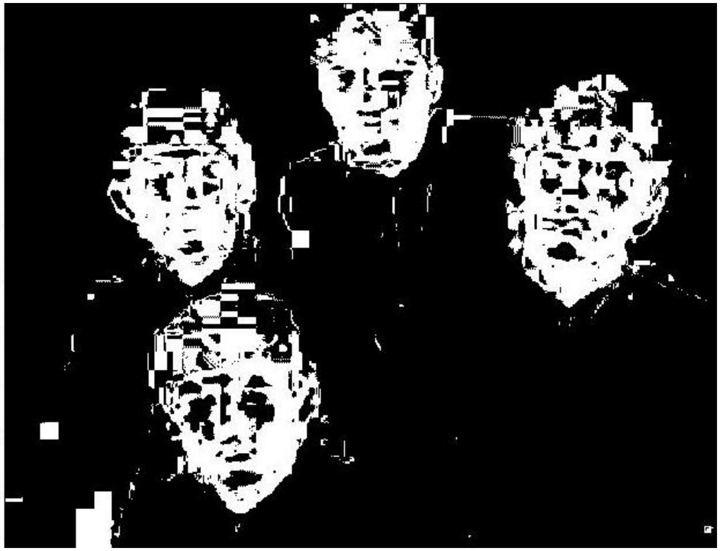
- **Looking for dark pixels…**



`fg_pix = find(im < 65);`

Uses: Color Based Detection

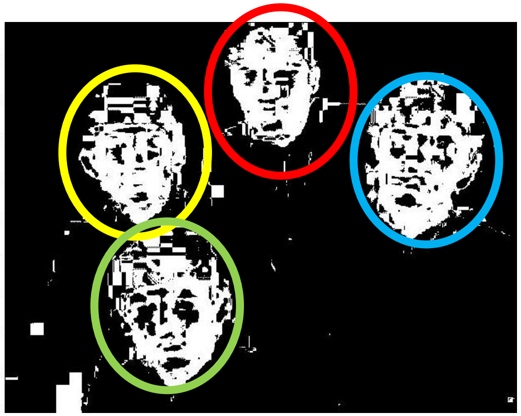
- Looking for pixels within a certain color range…



```
fg_pix = find(hue > t1 & hue < t2);
```

Issues

- **How to demarcate multiple regions of interest?**
  - Count objects
  - Compute further features per object



- **What to do with "noisy" binary outputs?**
  - Holes
  - Extra small fragments

Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- Extract individual objects
  - Connected Components Labeling
- Describe the objects
  - Region properties

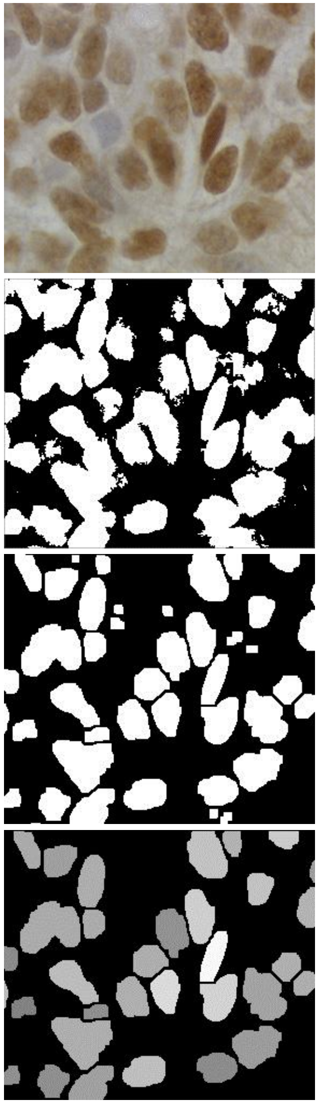


Image Source: D. Kim et al., Cytometry 35(1), 1999

Thresholding

- Grayscale image ⇒ Binary mask
- Different variants
  - One-sided

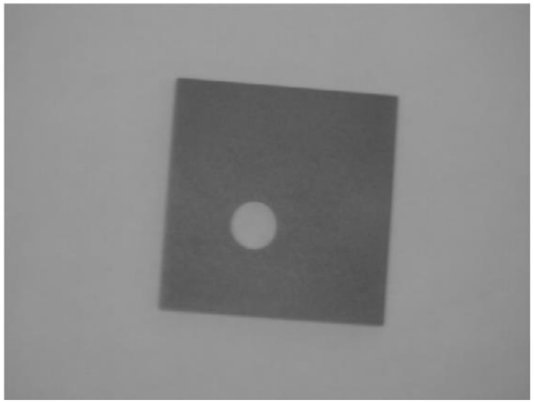
$$F_T[i,j] = \begin{cases} 1, & \text{if } F[i,j] \geq T \\ 0, & \text{otherwise} \end{cases}$$

  - Two-sided

$$F_T[i,j] = \begin{cases} 1, & \text{if } T_1 \leq F[i,j] \leq T_2 \\ 0, & \text{otherwise} \end{cases}$$

  - Set membership

$$F_T[i,j] = \begin{cases} 1, & \text{if } F[i,j] \in Z \\ 0, & \text{otherwise} \end{cases}$$



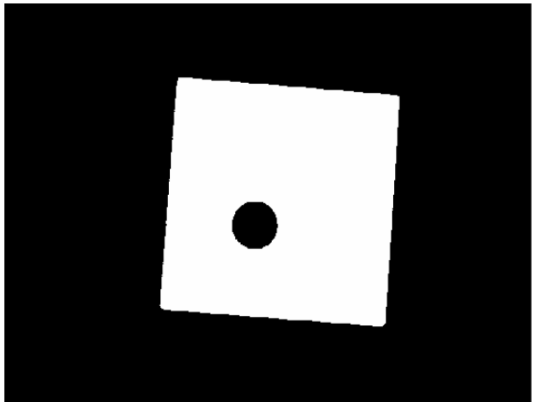
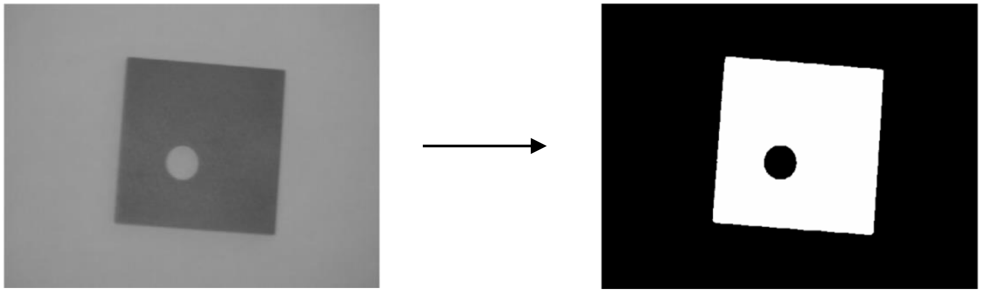


Image Source: http://homepages.inf.ed.ac.uk/rbf/HIPR2/

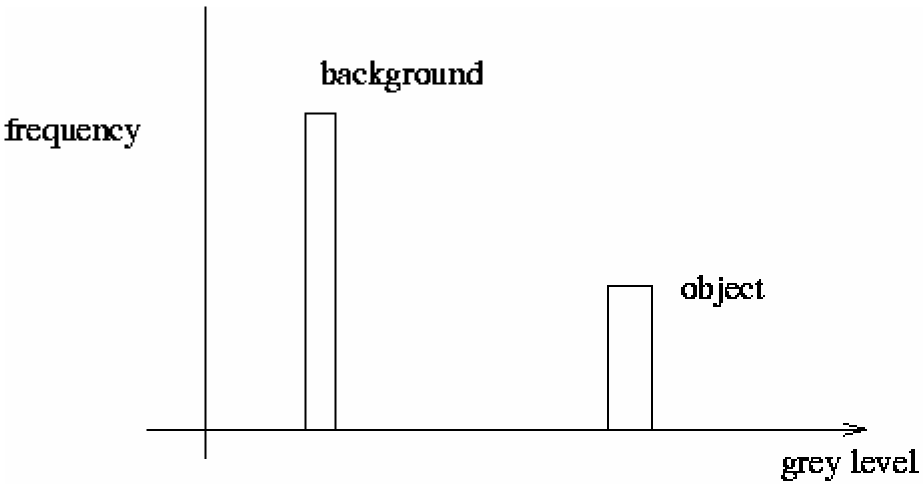
Selecting Thresholds

- Typical scenario
  - Separate an object from a distinct background

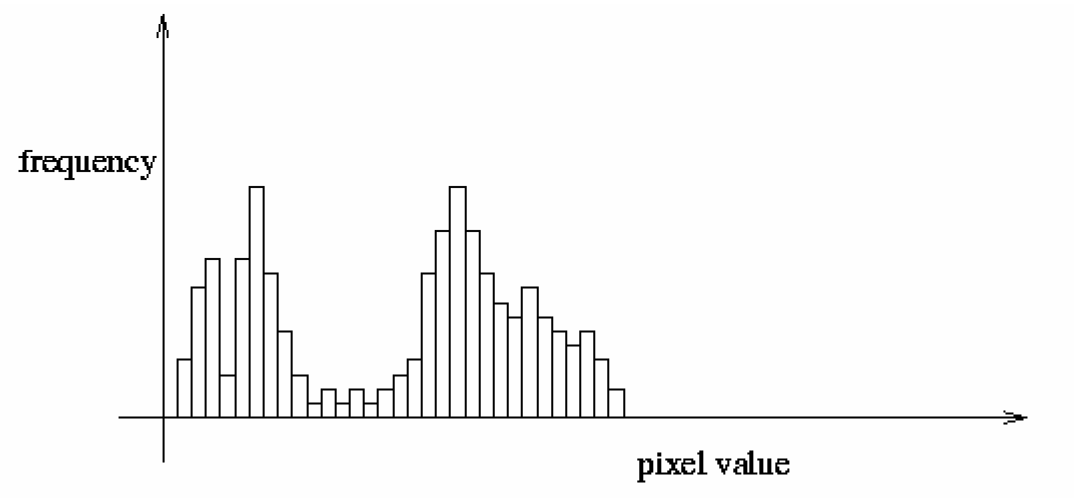


- Try to separate the different grayvalue distributions
  - Partition a bimodal histogram
  - Fit a parametric distribution (e.g. Mixture of Gaussians)
  - Dynamic or local thresholds
- In the following, I will present some simple methods.
  - We will then see some more general methods in Lecture 6…

A Nice Case: Bimodal Intensity Histograms



Ideal histogram, light object on dark background

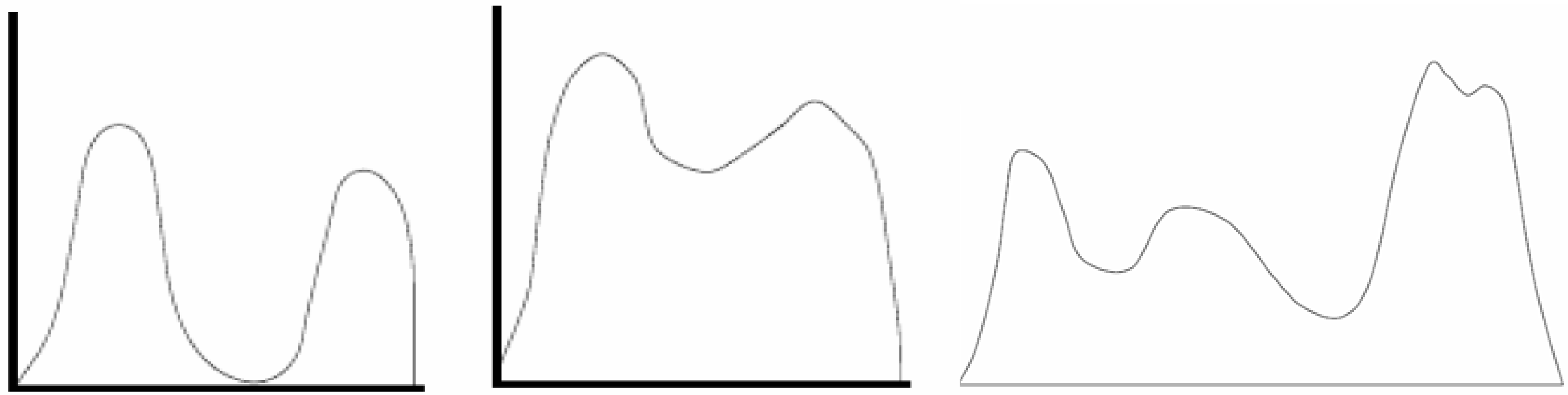


Actual observed histogram with noise

Source: Robyn Owens

Not so Nice Cases...

- How to separate those?



Two distinct modes | Overlapping modes | Multiple modes

- Threshold selection is difficult in the general case
  - Domain knowledge often helps
  - E.g. Fraction of text on a document page (⇒ histogram quantile)
  - E.g. Size of objects/structure elements

Source: Shapiro & Stockman

Global Binarization [Otsu'79]

- Search for the threshold T that minimizes the within-class variance σ_{within} of the two classes separated by T

$$\sigma^2_{within}(T) = n_1(T)\sigma_1^2 + n_2(T)\sigma_2^2(T)$$

where

$$n_1(T) = |\{I_{(x,y)} < T\}|, \quad n_2(T) = |\{I_{(x,y)} \geq T\}|$$

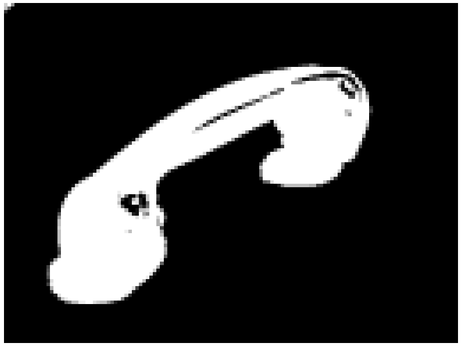
- This is the same as maximizing the between-class variance $\sigma_{between}$

$$\begin{aligned} \sigma^2_{between}(T) &= \sigma^2 - \sigma^2_{within}(T) \\ &= n_1(T)n_2(T)\left[\mu_1(T) - \mu_2(T)\right]^2 \end{aligned}$$

Algorithm

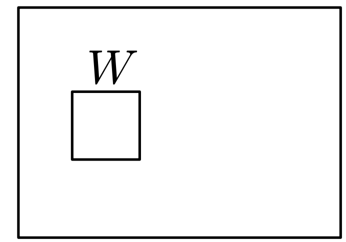
1. Precompute a cumulative grayvalue histogram h.
2. For each potential threshold T
   a) Separate the pixels into two clusters according to T
   b) Look up n_1, n_2 in h and compute both cluster means
   c) Compute $\sigma^2_{between}(T) = n_1(T)n_2(T)\left[\mu_1(T) - \mu_2(T)\right]^2$
3. Choose

$$T^* = \arg\max_T \left[\sigma^2_{between}(T)\right]$$



Local Binarization [Niblack'86]

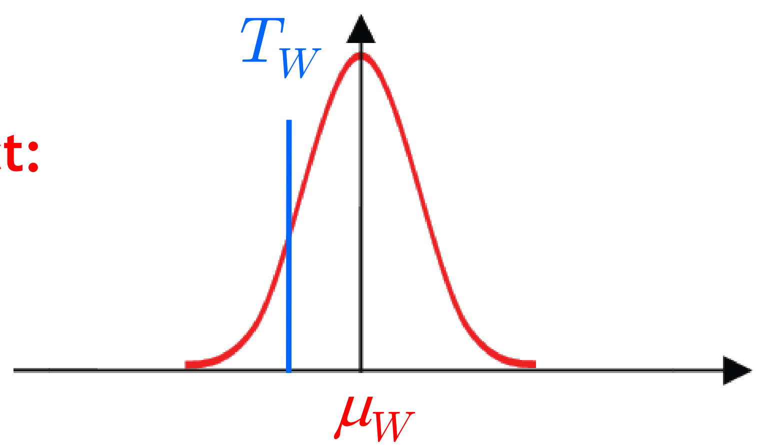
- **Estimate a local threshold within a small neighborhood window W**



$$T_W = \mu_W + k \cdot \sigma_W$$

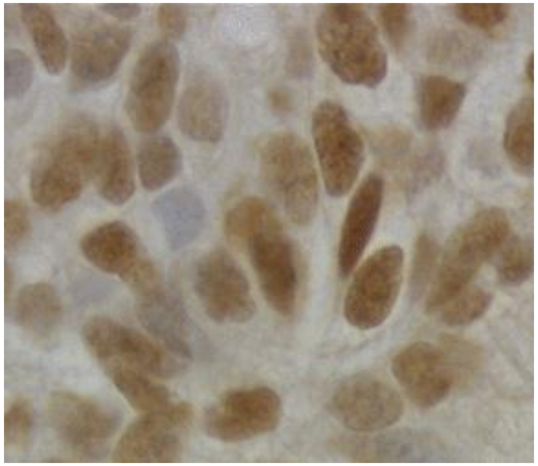
where $k \in [-1,0]$ is a user-defined parameter.

Effect:

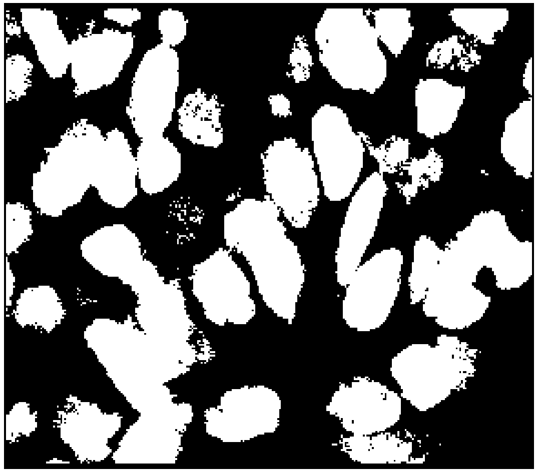


What is the hidden assumption here?

Effects



Original image



Global threshold selection (Otsu)

Local threshold selection (Niblack)

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Additional Improvements

- Document images often contain a smooth gradient

⇒ *Try to fit that gradient with a polynomial function*

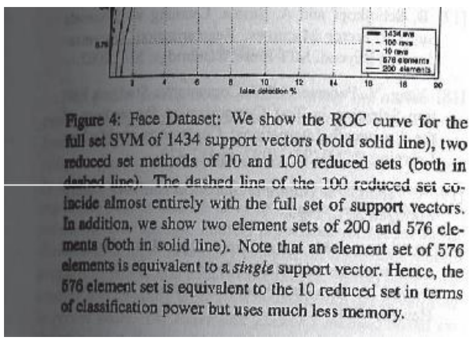
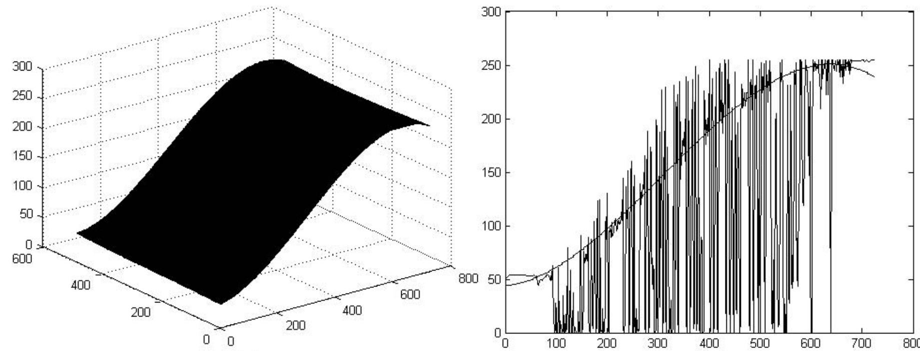
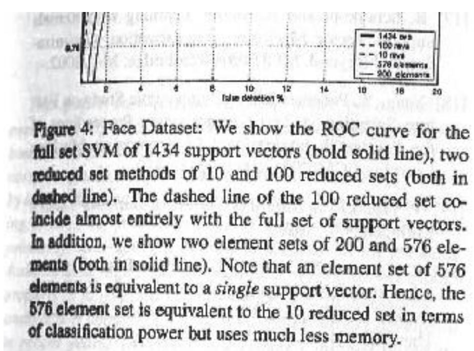


Figure 4: Face Dataset: We show the ROC curve for the full set SVM of 1434 support vectors (bold solid line), two reduced set methods of 10 and 100 reduced sets (both in dashed line). The dashed line of the 100 reduced set coincide almost entirely with the full set of support vectors. In addition, we show two element sets of 200 and 576 elements (both in solid line). Note that an element set of 576 elements is equivalent to a *single* support vector. Hence, the 576 element set is equivalent to the 10 reduced set in terms of classification power but uses much less memory.

Original image



Fitted surface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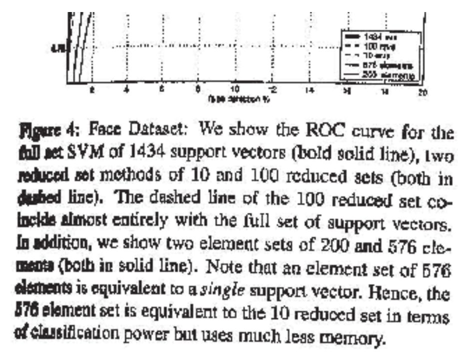
Shading compensation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Binarized result

Source: S. Lu & C. Tan, ICDAR'07

Polynomial Surface Fitting

- **Polynomial surface of degree d**

$$f(x,y) = \sum_{i+j=0}^{d} b_{i,j} x^i y^j$$

- **For an image pixel (x_0, y_0) with intensity I_0, this means**

$$b_{0,0} + b_{1,0}x_0 + b_{0,1}y_0 + b_{2,0}x_0^2 + b_{1,1,}x_0y_0 + \cdots + b_{0,3}y_0^3 = I_0$$

- **Least-squares estimation, e.g. for $d = 3$**

$$Ab = I$$

$$\begin{bmatrix} 1 & x_0 & y_0 & x_0^2 & x_0y_0 & \cdots & y_0^3 \\ 1 & x_1 & y_1 & x_1^2 & x_1y_1 & \cdots & y_1^3 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & y_n & x_n^2 & x_ny_n & \cdots & y_n^3 \end{bmatrix} \begin{bmatrix} b_{0,0} \\ b_{1,0} \\ \vdots \\ b_{0,3} \end{bmatrix} = \begin{bmatrix} I_0 \\ I_1 \\ \vdots \\ I_n \end{bmatrix}$$

Solution with pseudo-inverse:

$$b = (A^TA)^{-1}A^TI$$

Matlab (using SVD):

$$b = I \setminus A$$

Surface Fitting

- Iterative Algorithm
  1.) Fit parametric surface to all points in region.
  2.) Subtract estimated surface.
  3.) Apply global threshold (*e.g.* with Otsu method)

  Initial guess

  4.) Fit surface to all *background* pixels in original region.
  5.) Subtract estimated surface.
  6.) Apply global threshold (Otsu)

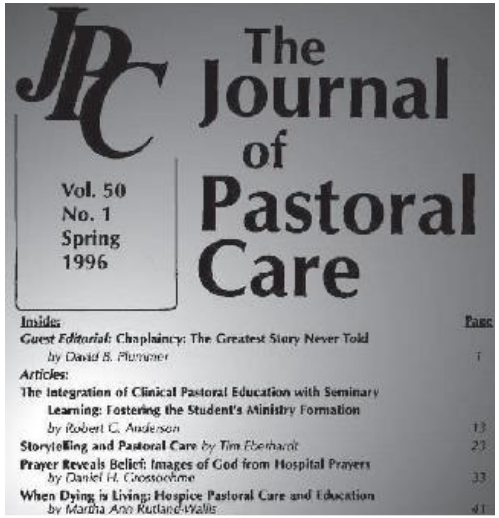
  Refined guess

  7.) *Iterate further if needed…*

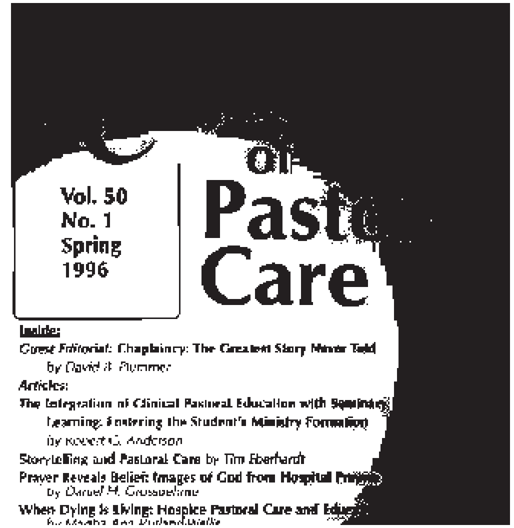
- The first pass also takes foreground pixels into account.
  - This is corrected in the following passes.
  - Basic assumption here: most pixels belong to the background.

Result Comparison

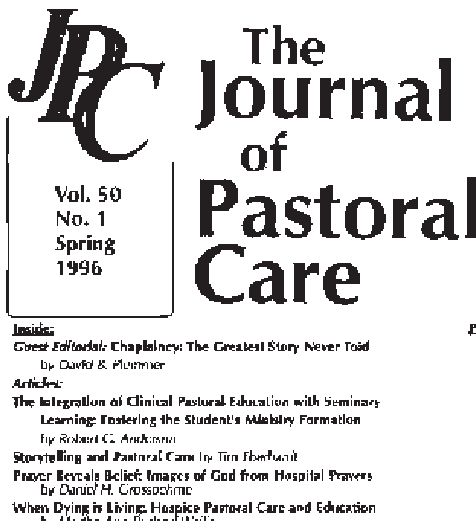
Original image



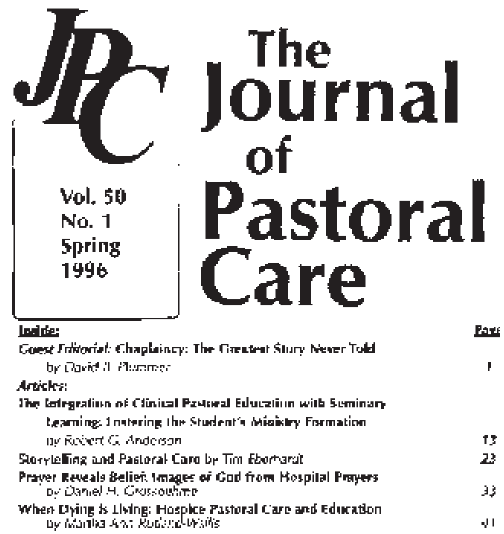
Global (Otsu)



Local (Niblack)



Polynomial + Global



Source: S. Lu & C. Tan, ICDAR'07

Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- Extract individual objects
  - Connected Components Labeling
- Describe the objects
  - Region properties

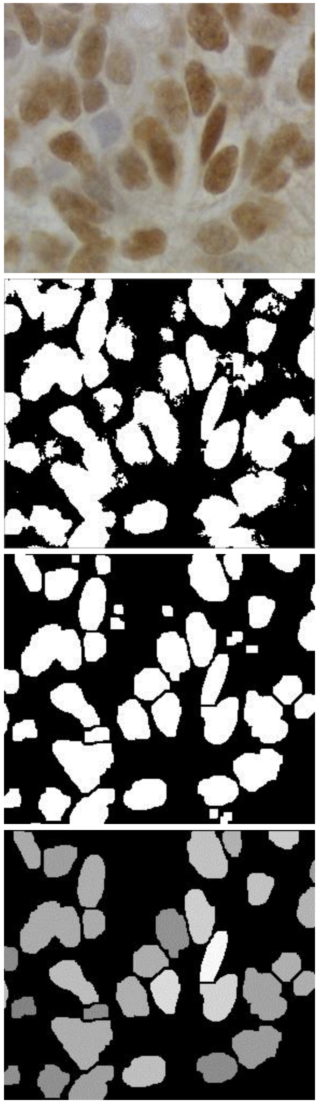
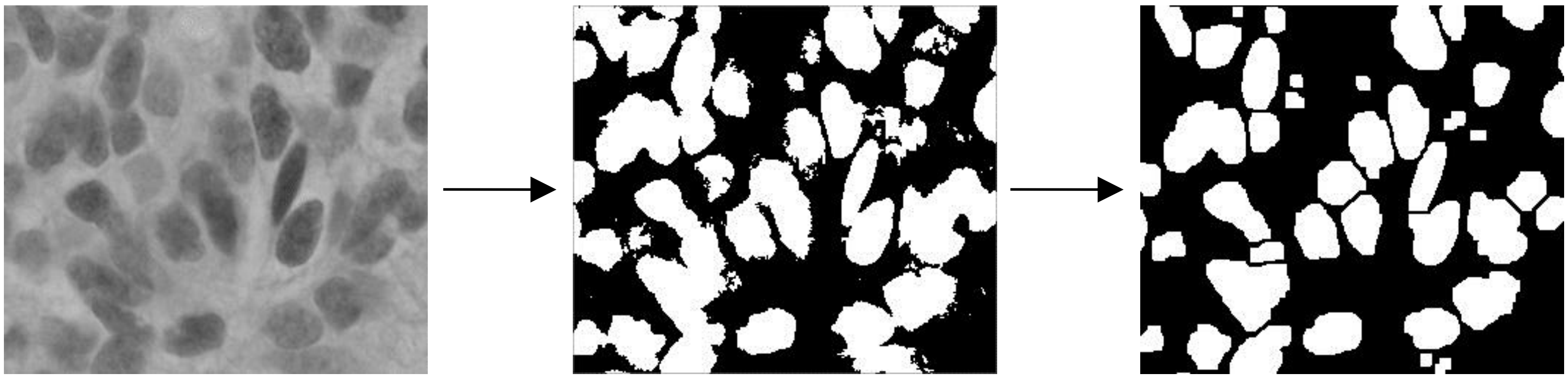


Image Source: D. Kim et al., Cytometry 35(1), 1999

Cleaning the Binarized Results

- Results of thresholding often still contain noise



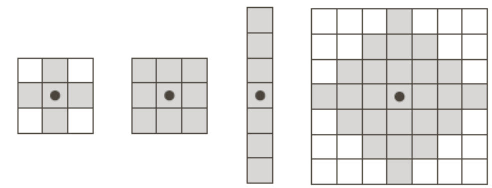
- Necessary cleaning operations
  - Remove isolated points and small structures
  - Fill holes

⇒ **Morphological Operators**

Image Source: D. Kim et al., Cytometry 35(1), 1999

Morphological Operators

- Basic idea
  - Scan the image with a structuring element
  - Perform set operations (intersection, union) of image content with structuring element



Matlab:
`>> help strel`

- Two basic operations
  - Dilation (Matlab: `imdilate`)
  - Erosion (Matlab: `imerode`)
- Several important combinations
  - Opening (Matlab: `imopen`)
  - Closing (Matlab: `imclose`)
  - Boundary extraction

Image Source: R.C. Gonzales & R.E. Woods

Dilation

- **Definition**
  - *"The dilation of A by B is the set of all displacements z, such that $(\hat{B})_z$ and A overlap by at least one element".*
  - ($(\hat{B})_z$ is the mirrored version of B, shifted by z)
- **Effects**
  - If current pixel z is foreground, set all pixels under $(B)_z$ to foreground.

  ⇒ Expand connected components

  ⇒ Grow features

  ⇒ Fill holes

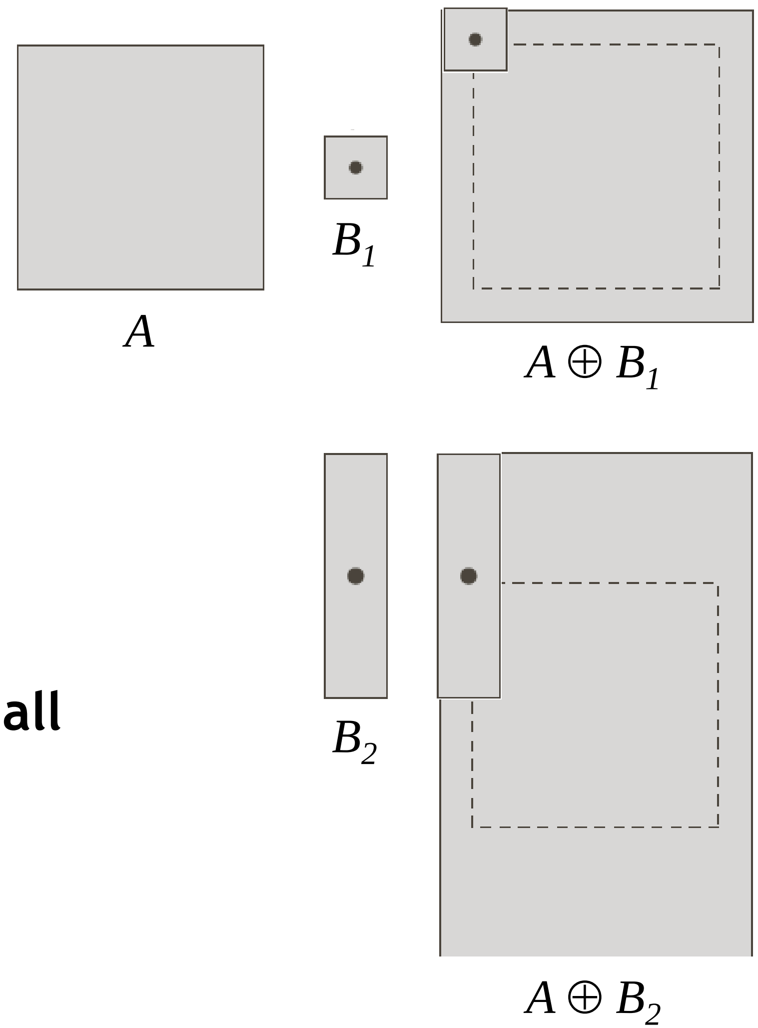


Image Source: R.C. Gonzales & R.E. Woods

Erosion

- **Definition**
  - *"The erosion of A by B is the set of all displacements z, such that $(B)_z$ is entirely contained in A".*

- **Effects**
  - If not every pixel under $(B)_z$ is foreground, set the current pixel z to background.

  $\Rightarrow$ Erode connected components

  $\Rightarrow$ Shrink features

  $\Rightarrow$ Remove bridges, branches, noise

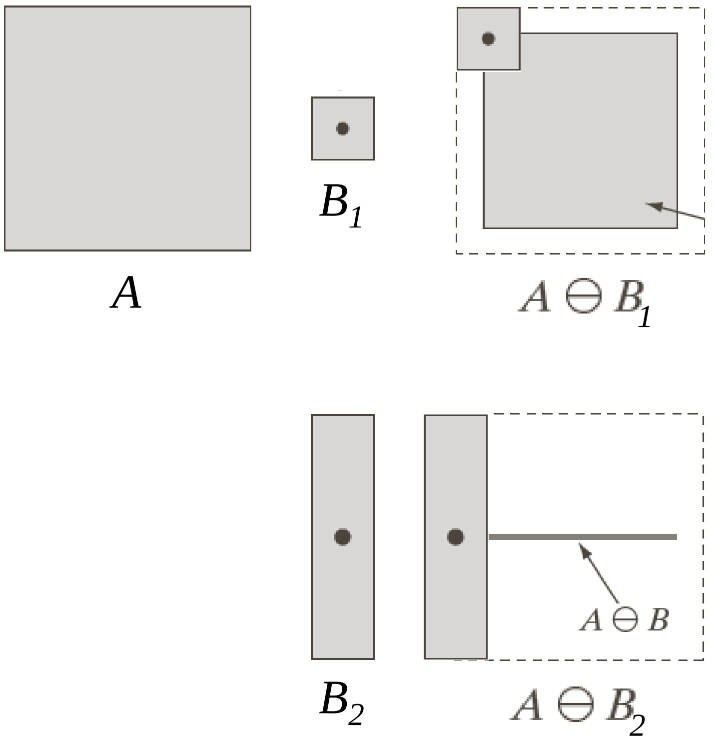
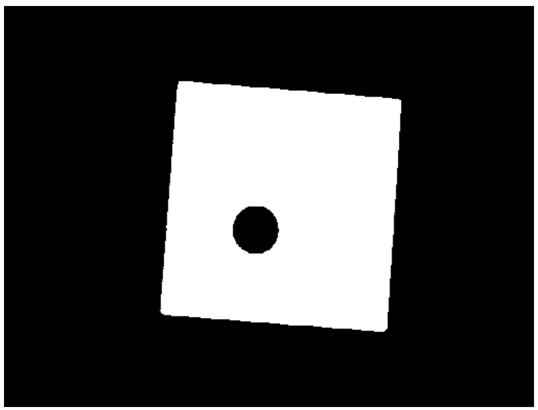
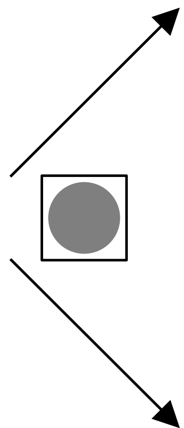


Image Source: R.C. Gonzales & R.E. Woods

Effects



Original image

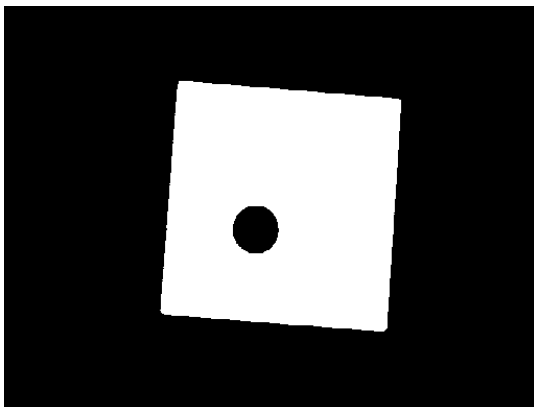


Dilation with circular structuring element

Erosion with circular structuring element

Image Source: http://homepages.inf.ed.ac.uk/rbf/HIPR2/

Effects



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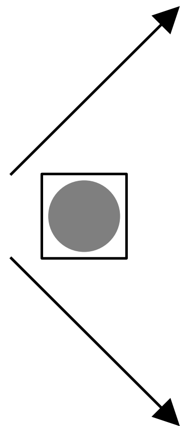
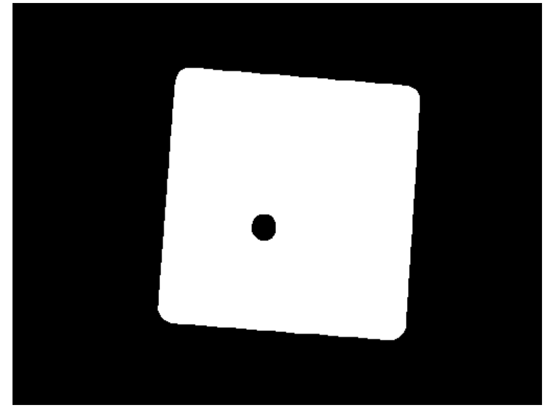
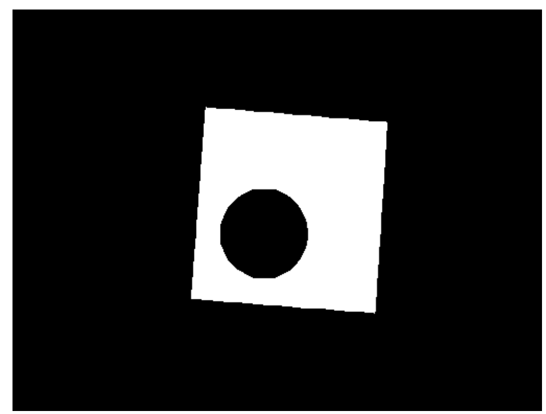




Dilation with circular structuring element



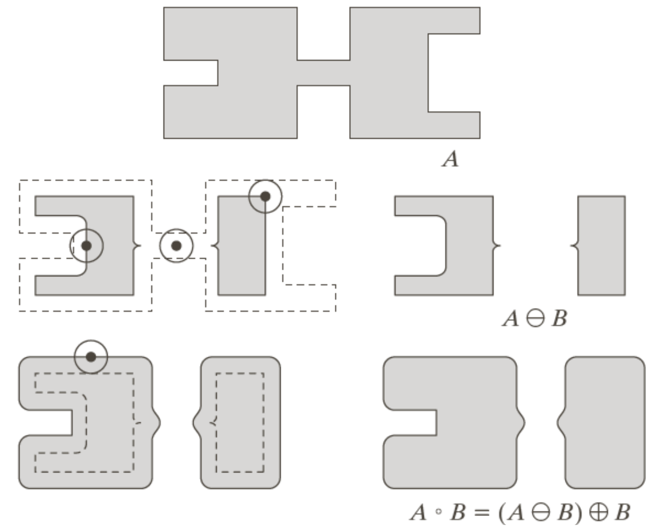
Erosion with circular structuring element

Image Source: http://homepages.inf.ed.ac.uk/rbf/HIPR2/

Opening

- **Definition**
  - Sequence of **Erosion** and **Dilation**

    $$A \circ B = (A \ominus B) \oplus B$$



- **Effect**
  - $A \circ B$ is defined by the points that are reached if B is *rolled around inside* A.

  $\Rightarrow$ Remove small objects, keep original shape.

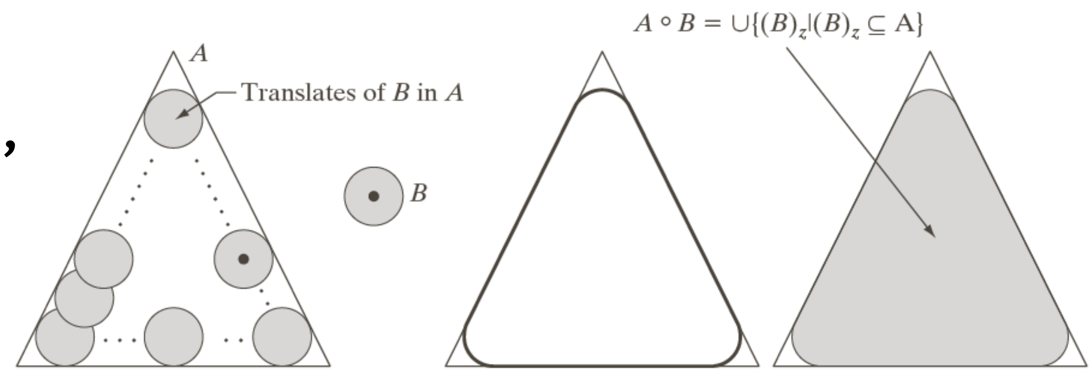
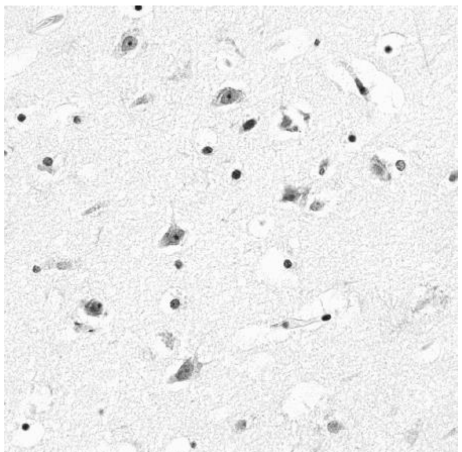


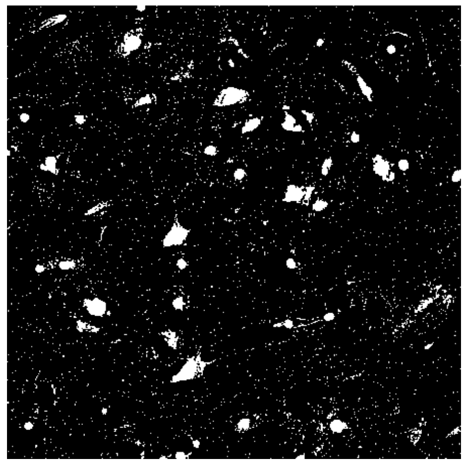
Image Source: R.C. Gonzales & R.E. Woods

Effect of Opening

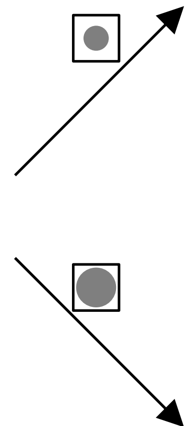
- Feature selection through ***size*** of structuring element

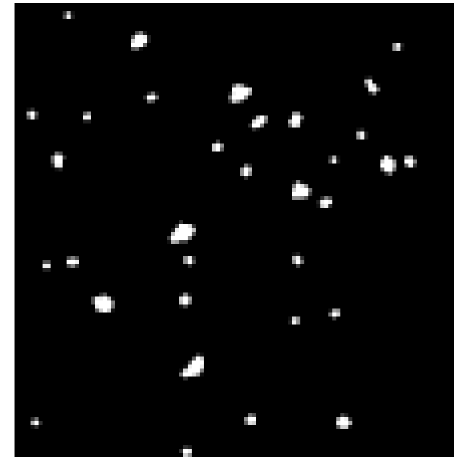


Original image

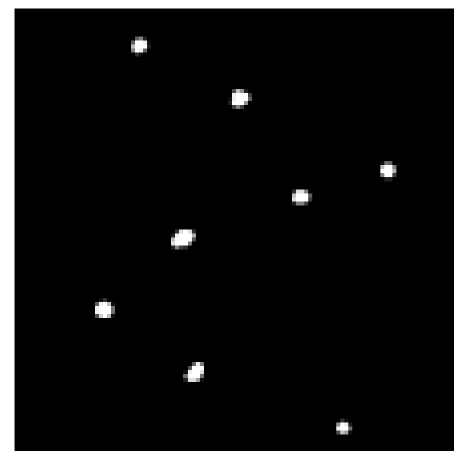


Thresholded





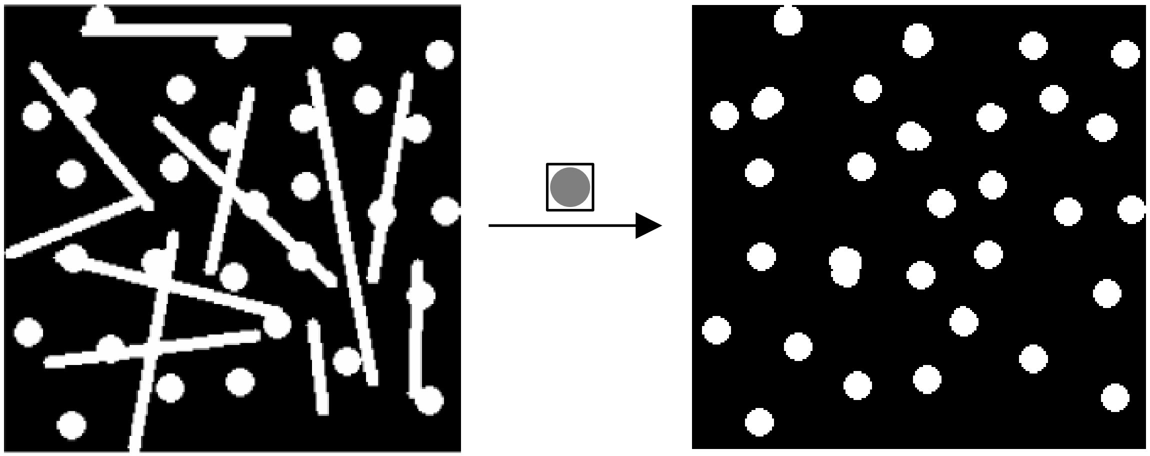
Opening with small structuring element



Opening with larger structuring element

Effect of Opening

- Feature selection through *shape* of structuring element



Input Image

Opening with circular structuring element

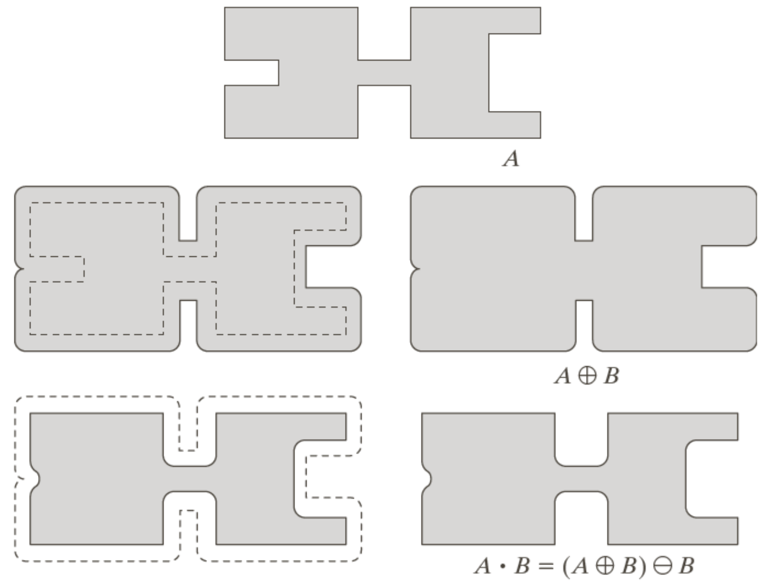
- *How could we have extracted the lines?*

Image Source: http://homepages.inf.ed.ac.uk/rbf/HIPR2/

Closing

- **Definition**
  - Sequence of **Dilation** and **Erosion**

    $$A \bullet B = (A \oplus B) \ominus B$$



- **Effect**
  - $A \bullet B$ is defined by the points that are reached if B is *rolled around on the outside* of A.

  $\Rightarrow$ Fill holes,
  keep original shape.

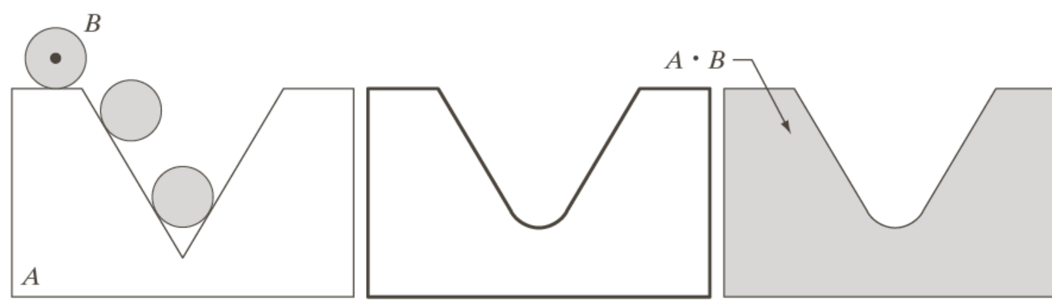
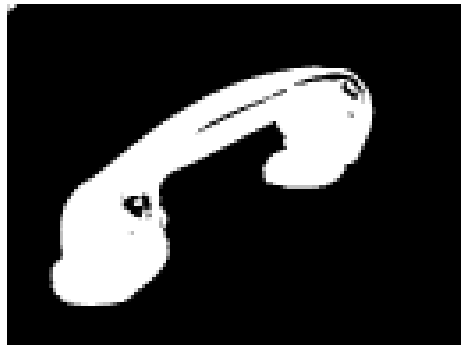


Image Source: R.C. Gonzales & R.E. Woods

Effect of Closing

- **Fill holes in thresholded image (*e.g.* due to specularities)**

Original image



Thresholded

Closing with circular structuring element

Size of structuring element determines which structures are selectively filled.

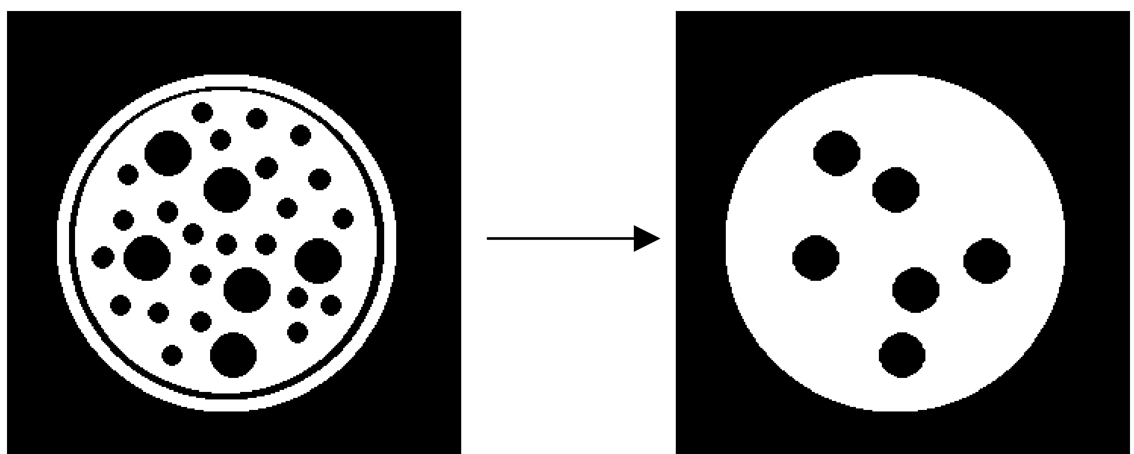
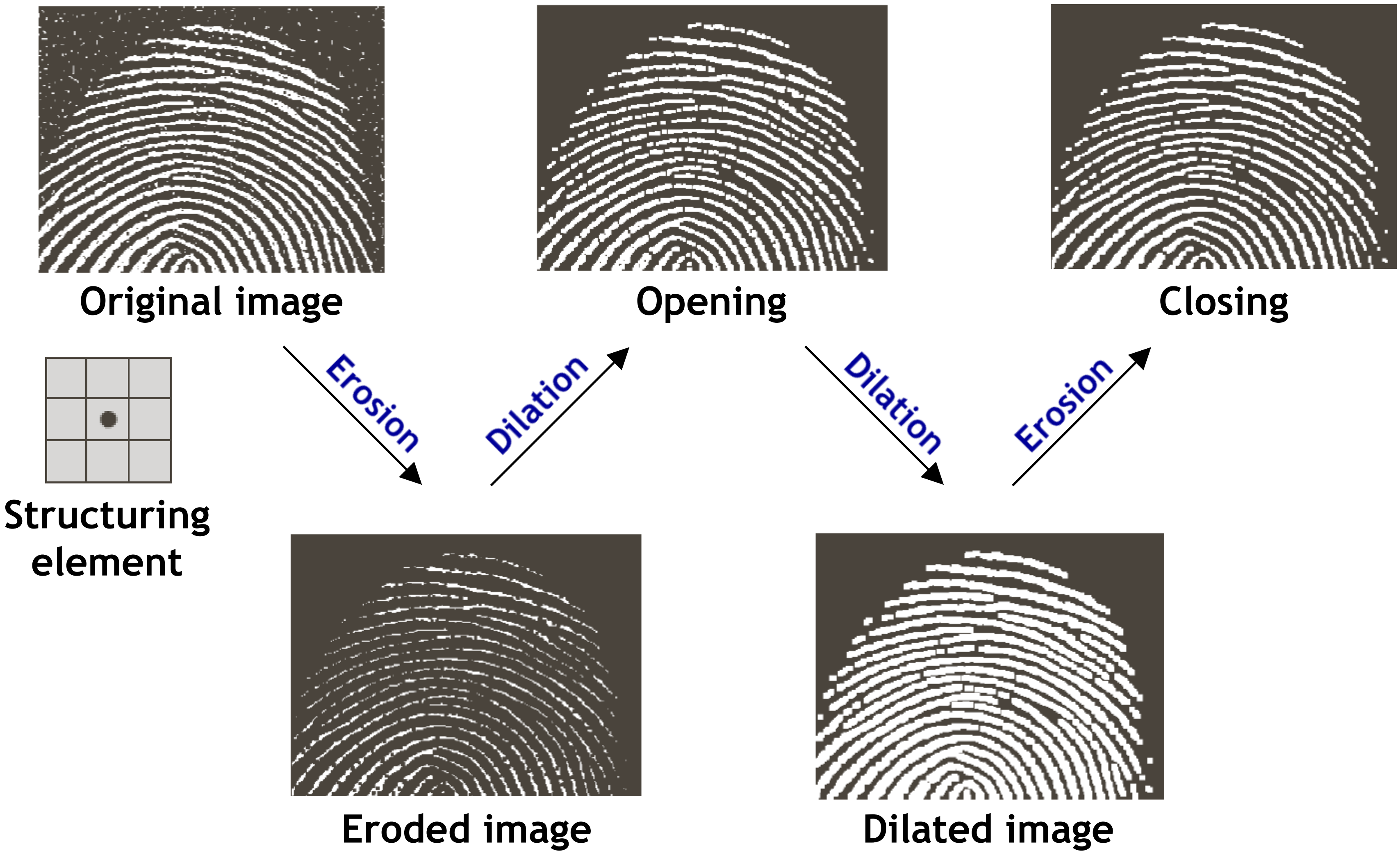


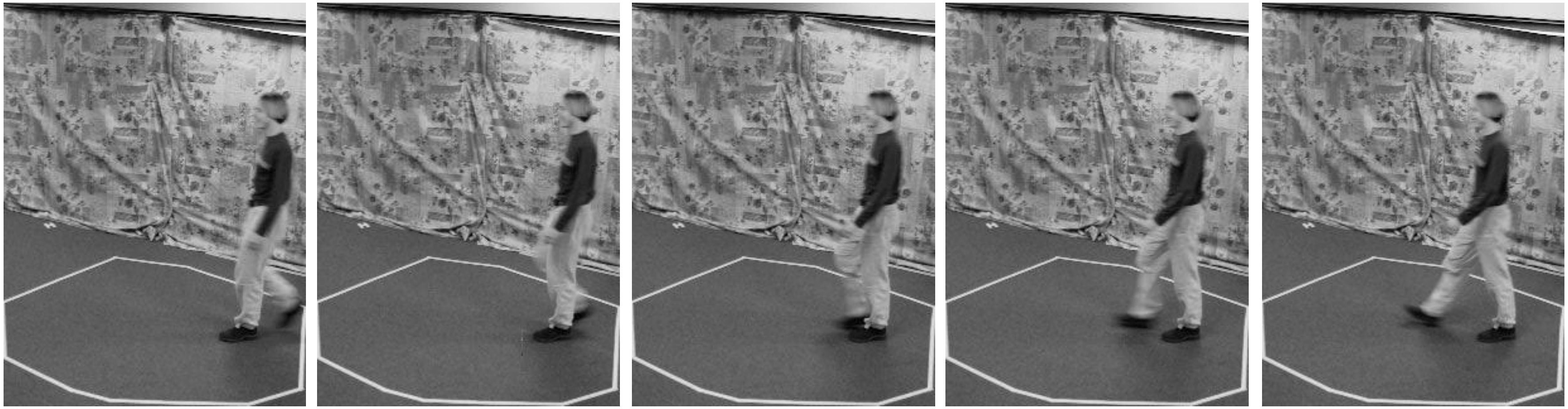
Image Source: http://homepages.inf.ed.ac.uk/rbf/HIPR2/

Example Application: Opening + Closing

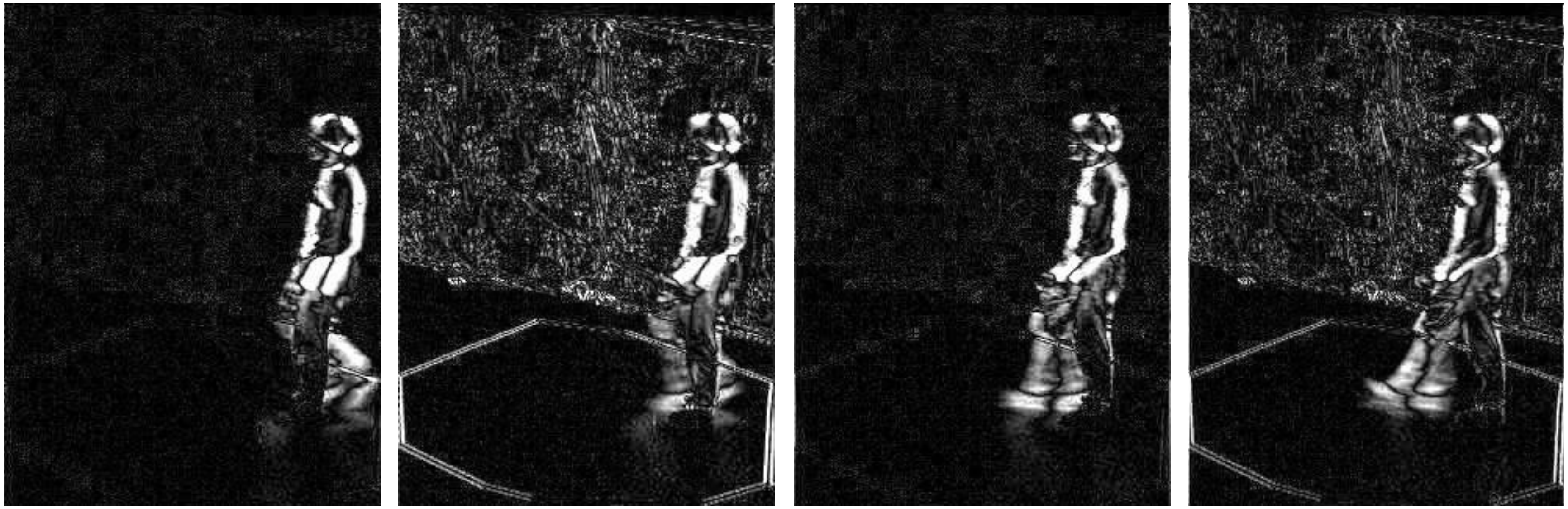


Source: R.C. Gonzales & R.E. Woods

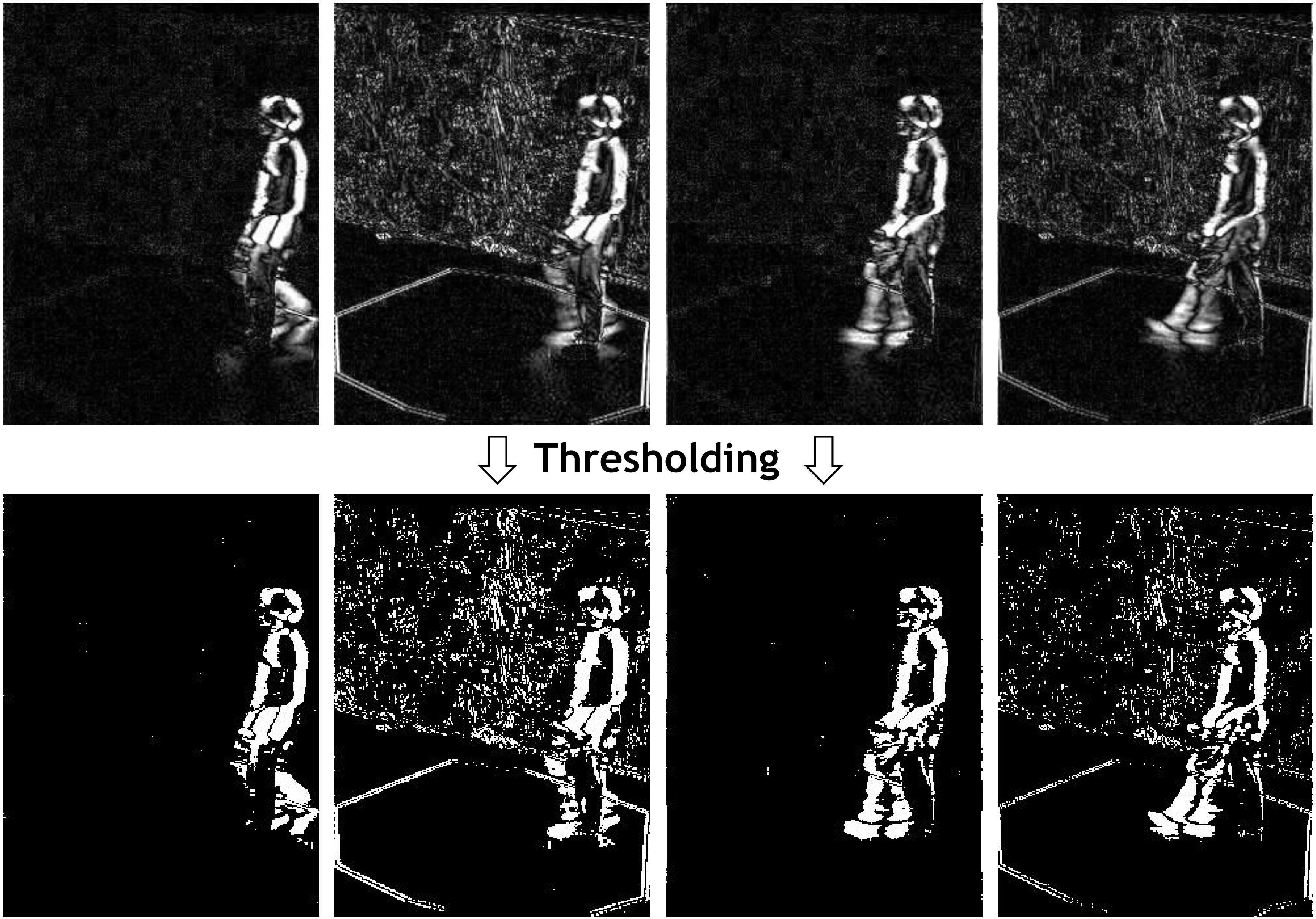
Application: Blob Tracking

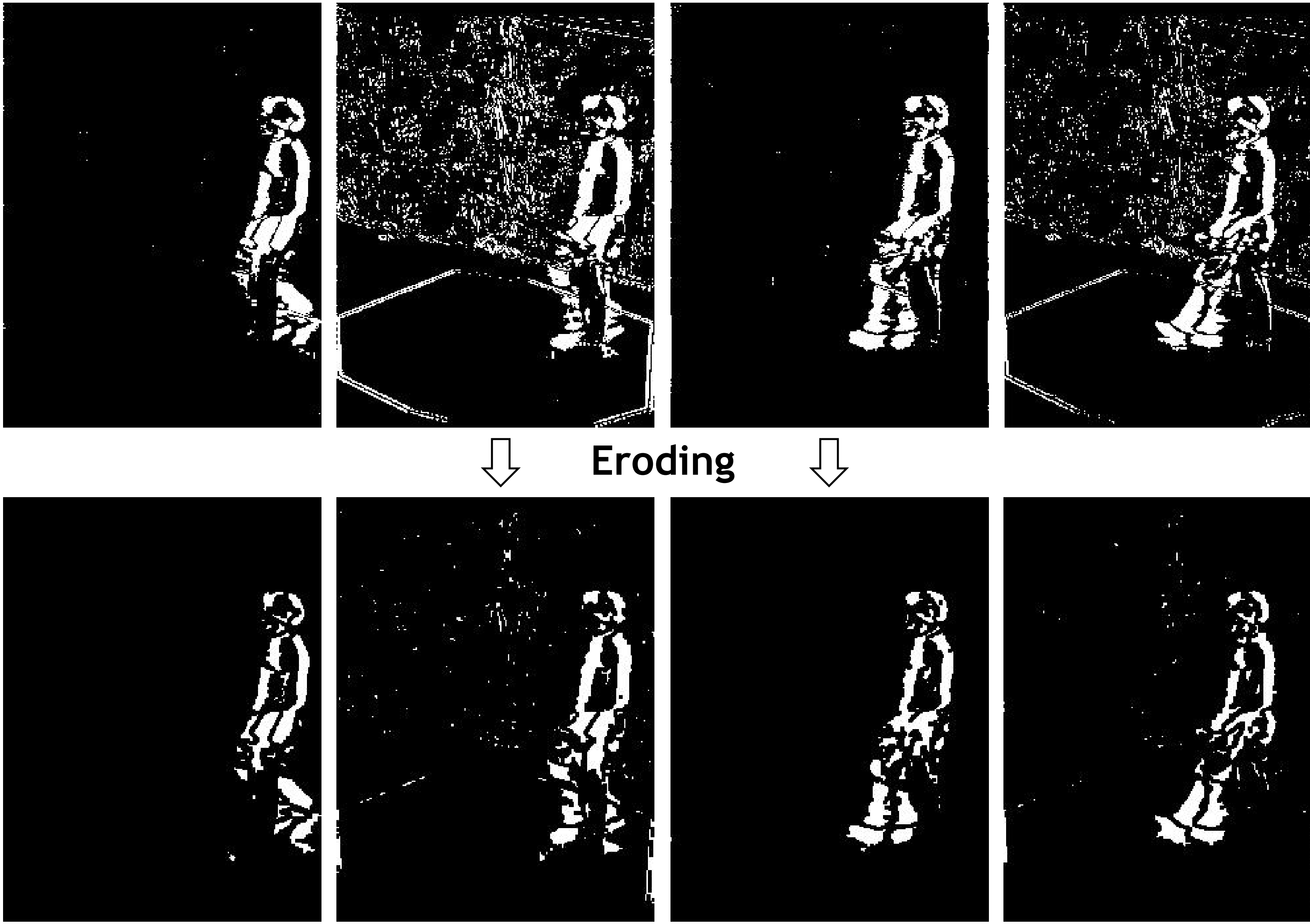


⇩ Absolute differences from frame to frame ⇩



Slide credit: K. Grauman


Thresholding


Eroding

Morphological Boundary Extraction

- **Definition**
  - First erode A by B, then subtract the result from the original A.

    $$\beta(A) = A - (A \ominus B)$$

- **Effects**
  - If a 3×3 structuring element is used, this results in a boundary that is exactly 1 pixel thick.

Source: R.C. Gonzales & R.E. Woods

Morphology Operators on Grayscale Images

- Sidenote
  - Dilation and erosion are typically performed on binary images.
  - If image is grayscale: for dilation take the neighborhood max, for erosion take the min.

Original Dilated Eroded

Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- **Extract individual objects**
  - Connected Components Labeling
- Describe the objects
  - Region properties

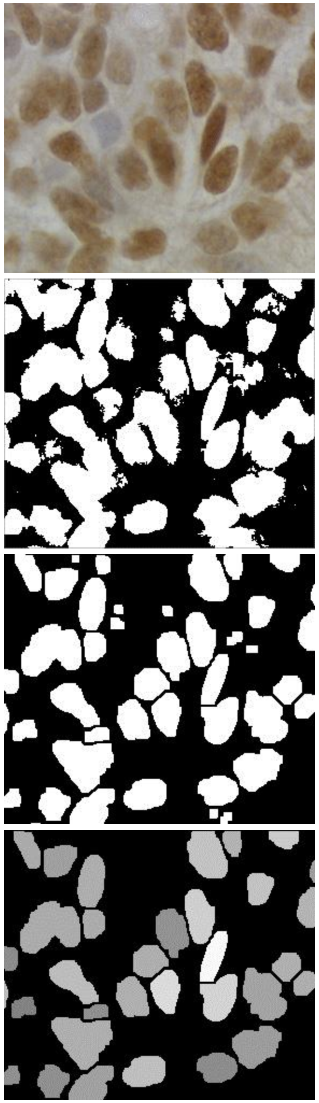
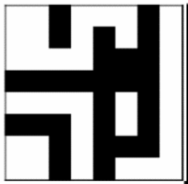


Image Source: D. Kim et al., Cytometry 35(1), 1999

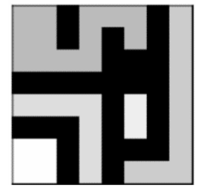
Connected Components Labeling

- **Goal: Identify distinct regions**



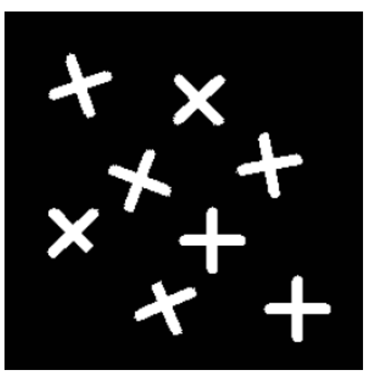
1	1	0	1	1	1	0	1
1	1	0	1	0	1	0	1
1	1	1	1	0	0	0	1
0	0	0	0	0	0	0	1
1	1	1	1	0	1	0	1
0	0	0	1	0	1	0	1
1	1	0	1	0	0	0	1
1	1	0	1	0	1	1	1

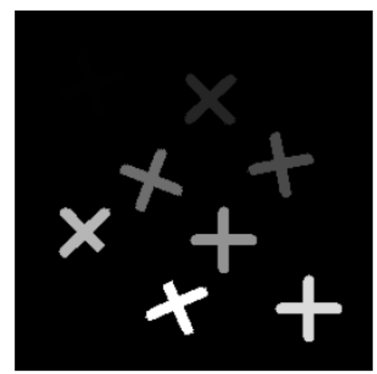
Binary image



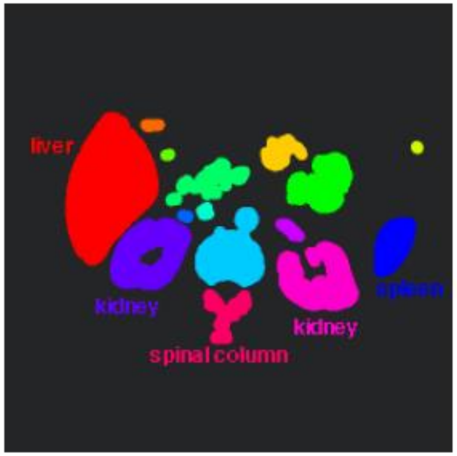
1	1	0	1	1	1	0	2
1	1	0	1	0	1	0	2
1	1	1	1	0	0	0	2
0	0	0	0	0	0	0	2
3	3	3	3	0	4	0	2
0	0	0	3	0	4	0	2
5	5	0	3	0	0	0	2
5	5	0	3	0	2	2	2

Connected components labeling





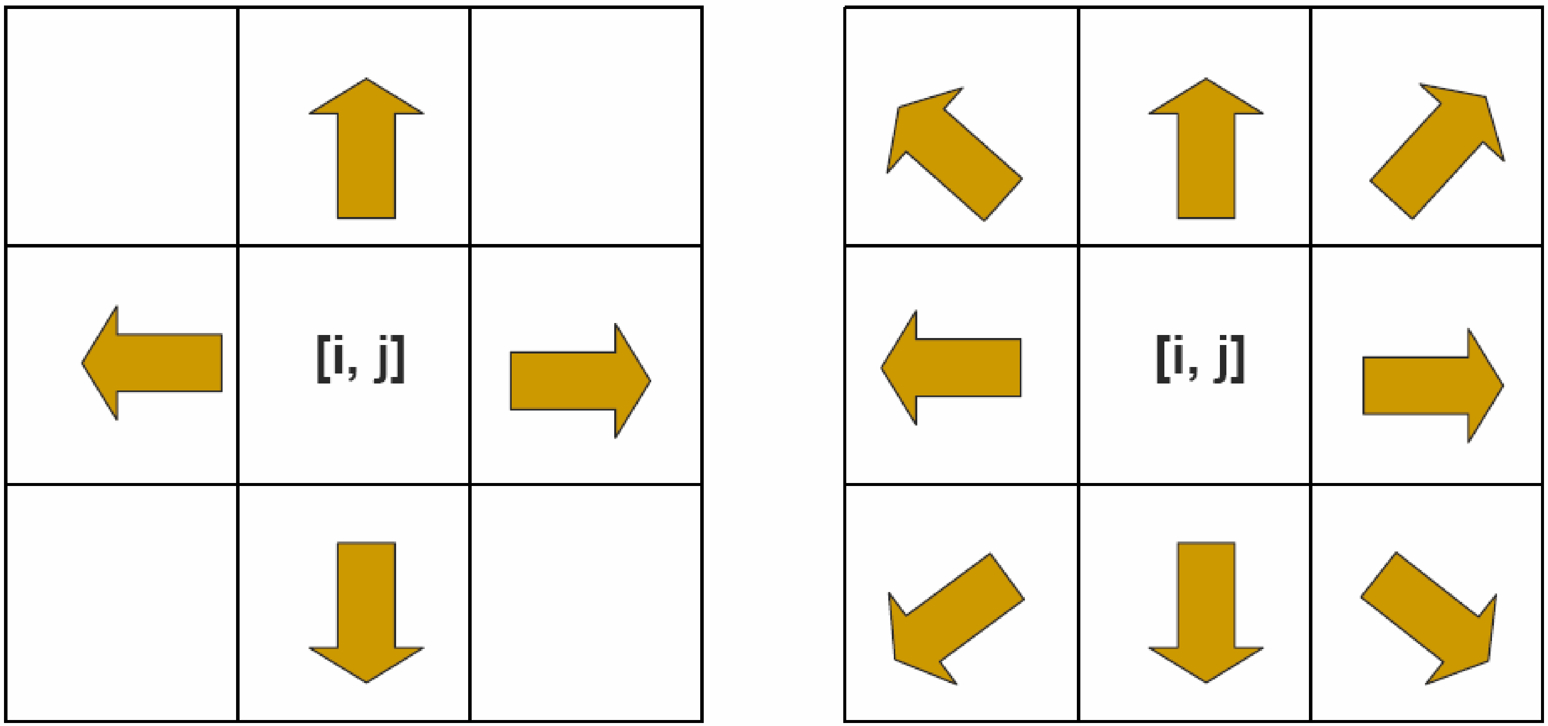
Connected Components Example



connected components of 1's from thresholded image

Connectedness

- **Which pixels are considered neighbors?**



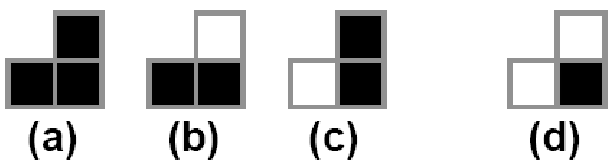
4-connected **8-connected**

Sequential Connected Components

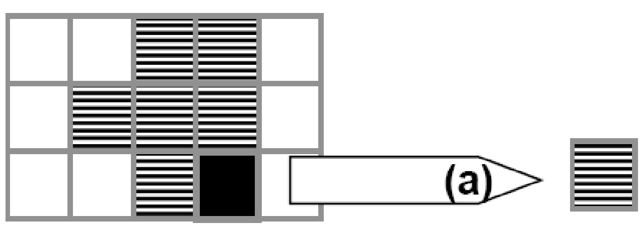
- Labeling a pixel only requires to consider its prior and superior neighbors.
- It depends on the type of connectivity used for foreground (4-connectivity here).

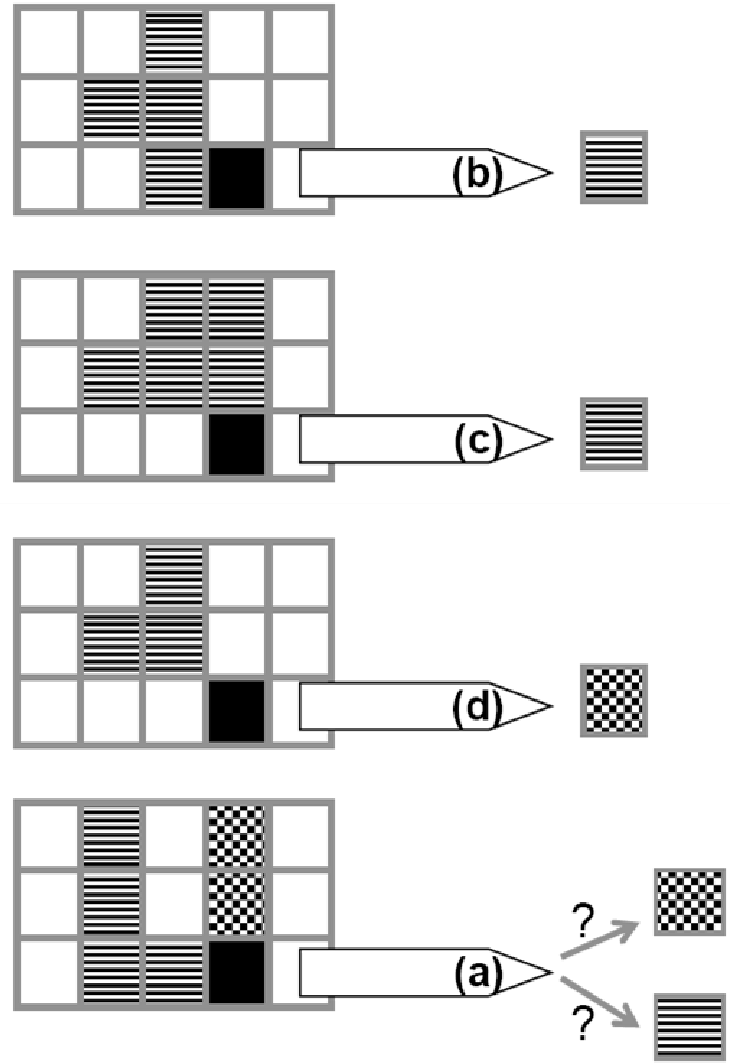
Same object

New object



What happens in these cases?



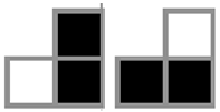


Equivalence table

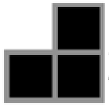
Sequential Connected Components (2)

- **Process the image from left to right, top to bottom:**

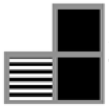
  1.) If the next pixel to process is 1



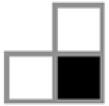
i.) If only one of its neighbors (top or left) is 1, copy its label.



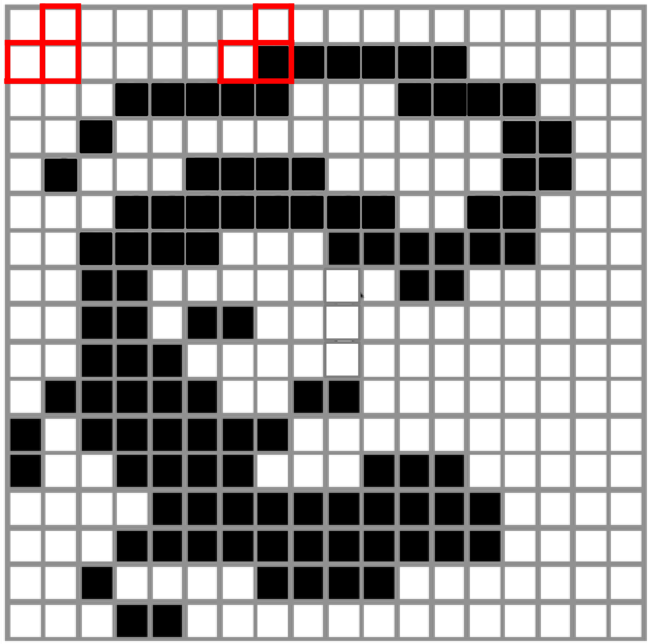
ii.) If both are 1 and have the same label, copy it.



iii.) If they have different labels
- Copy the label from the left.
- Update the equivalence table.



iv.) Otherwise, assign a new label.

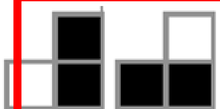


Equivalence table

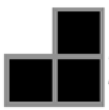
{1}

Sequential Connected Components (2)

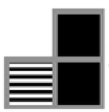
- Process the image from left to right, top to bottom:
  1.) If the next pixel to process is 1



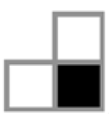
i.) If only one of its neighbors (top or left) is 1, copy its label.



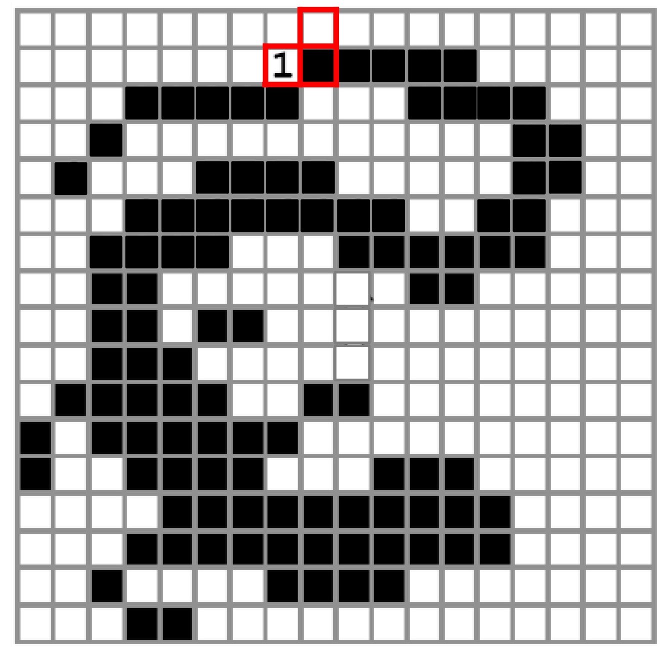
ii.) If both are 1 and have the same label, copy it.



iii.) If they have different labels
- Copy the label from the left.
- Update the equivalence table.



iv.) Otherwise, assign a new label.

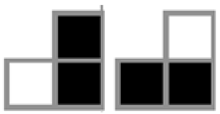


Equivalence table

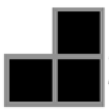
{1}

Sequential Connected Components (2)

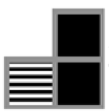
- **Process the image from left to right, top to bottom:**
  - 1.) If the next pixel to process is 1



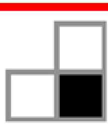
i.) If only one of its neighbors (top or left) is 1, copy its label.



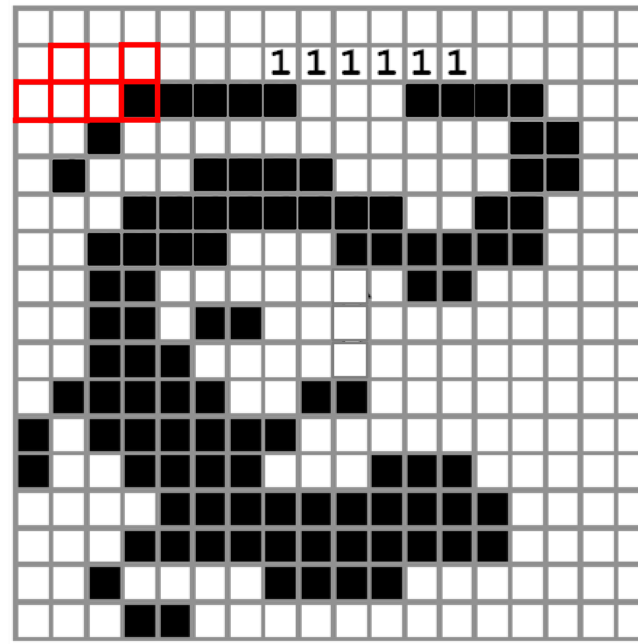
ii.) If both are 1 and have the same label, copy it.



iii.) If they have different labels
   – Copy the label from the left.
   – Update the equivalence table.



iv.) Otherwise, assign a new label.

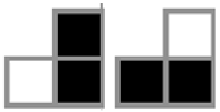


Equivalence table

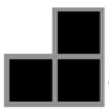
{1}
{2}

Sequential Connected Components (2)

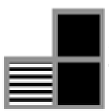
- **Process the image from left to right, top to bottom:**
  1.) If the next pixel to process is 1



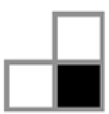
  i.) If only one of its neighbors (top or left) is 1, copy its label.



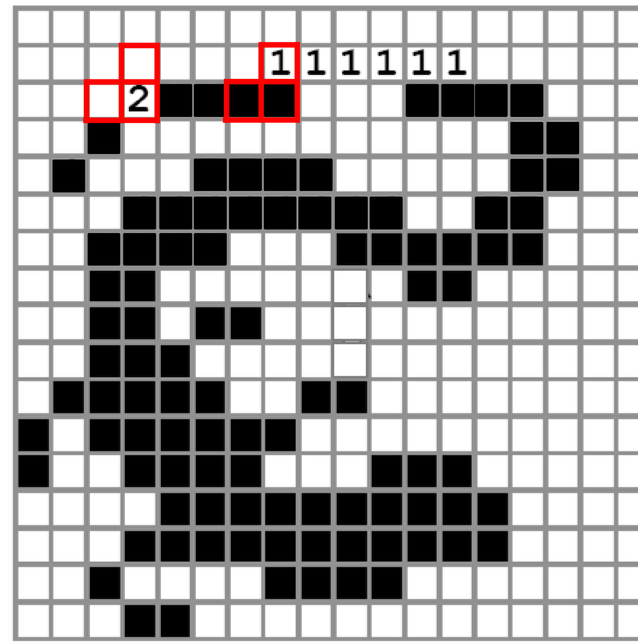
  ii.) If both are 1 and have the same label, copy it.



  iii.) If they have different labels
  - Copy the label from the left.
  - Update the equivalence table.



  iv.) Otherwise, assign a new label.

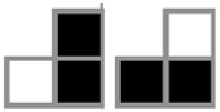


Equivalence table

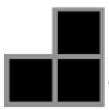
{1} 2}
{2}

Sequential Connected Components (2)

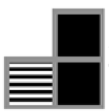
- **Process the image from left to right, top to bottom:**
  - 1.) If the next pixel to process is 1



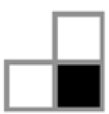
  - i.) If only one of its neighbors (top or left) is 1, copy its label.



  - ii.) If both are 1 and have the same label, copy it.

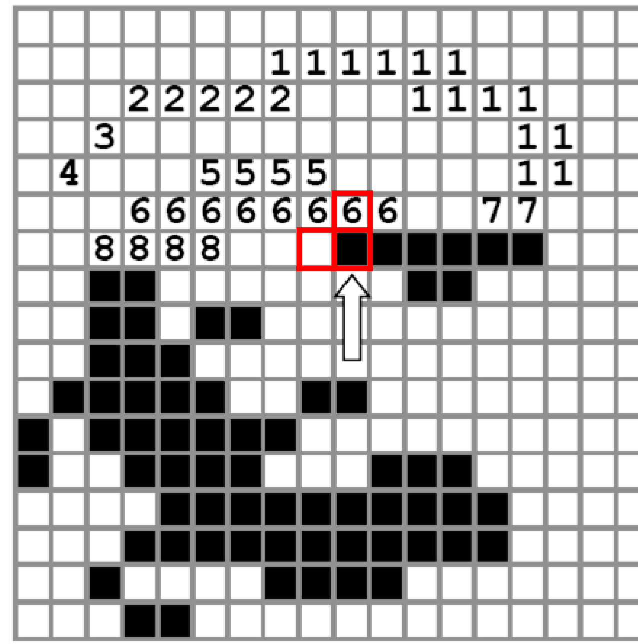


  - iii.) If they have different labels
    - – Copy the label from the left.
    - – Update the equivalence table.



  - iv.) Otherwise, assign a new label.

- **Re-label with the smallest of equivalent labels**



Equivalence table

{1, 2, 7}
{3}
{4}
{5, 6, 8}
{}

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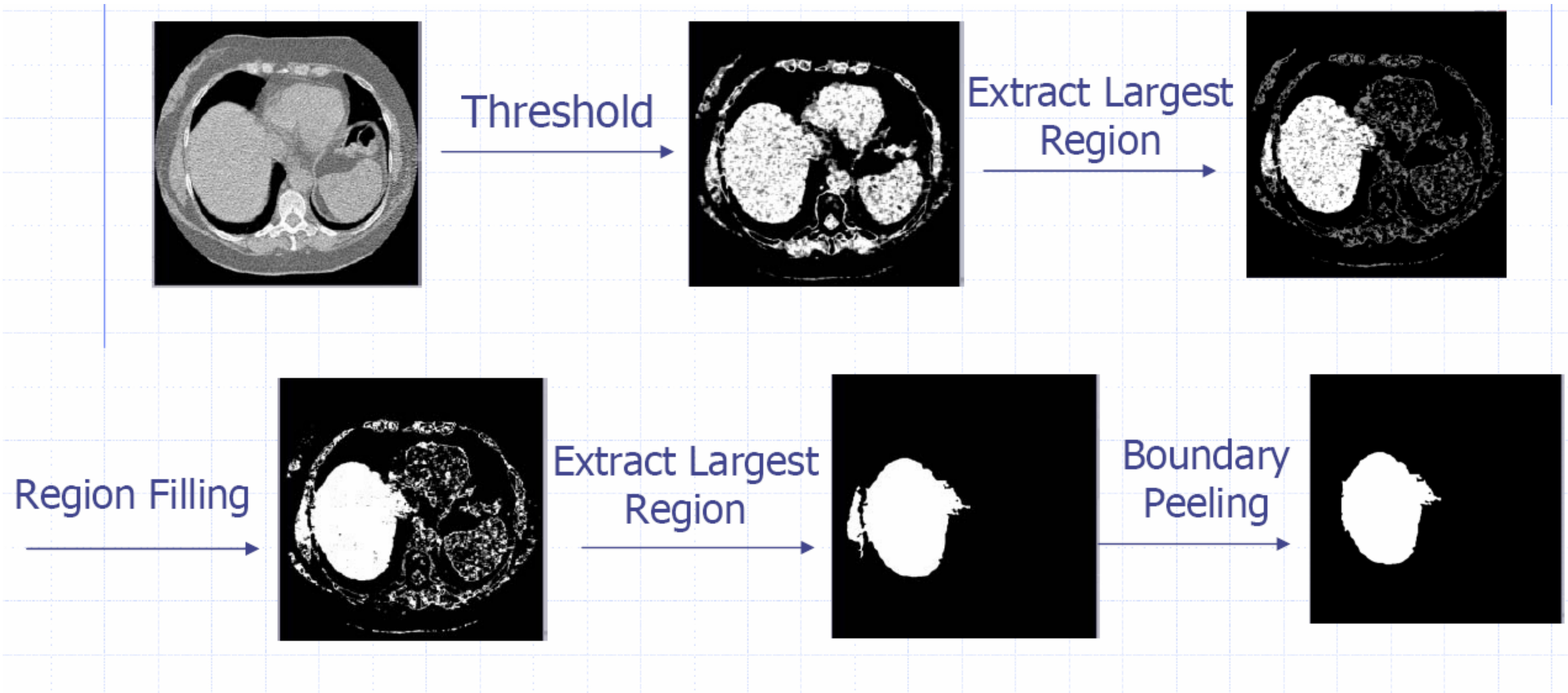
Application: Segmentation of a Liver



Application by Jie Zhu, Cornell University

Outline of Today's Lecture

- Convert the image into binary form
  - Thresholding
- Clean up the thresholded image
  - Morphological operators
- Extract individual objects
  - Connected Components Labeling
- **Describe the objects**
  - Region properties

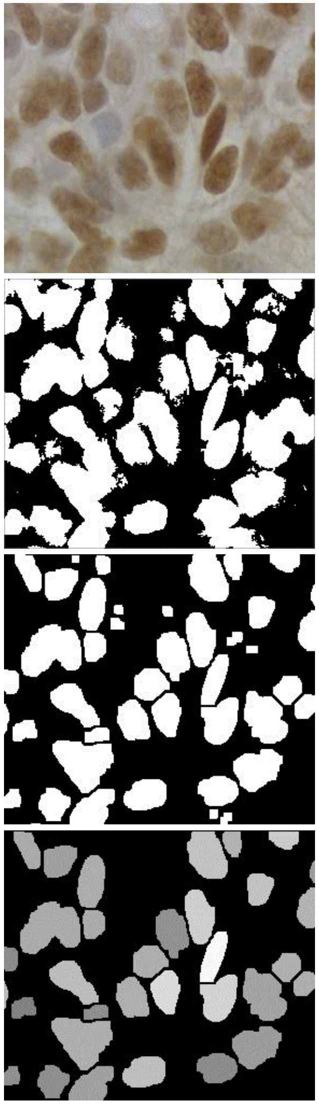


Image Source: D. Kim et al., Cytometry 35(1), 1999

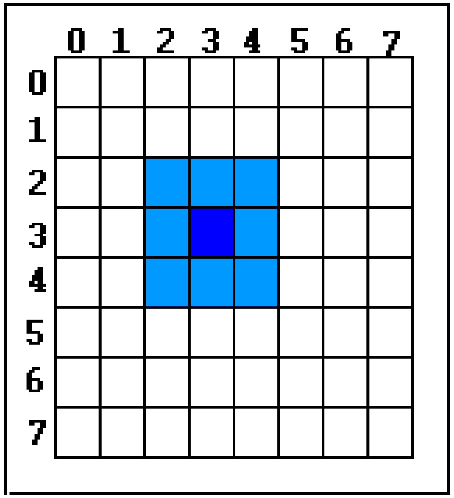
Region Properties

- From the previous steps, we can obtain separated objects.

- Some useful features can be extracted once we have connected components, including
  - Area
  - Centroid
  - Extremal points, bounding box
  - Circularity
  - Spatial moments

Area and Centroid

- We denote the set of pixels in a region by R
- Assuming square pixels, we obtain
  - *Area:*
    $$A = \sum_{(x,y)\in R} 1$$
  - *Centroid:*
    $$\bar{x} = \frac{1}{A} \sum_{(x,y)\in R} x$$
    $$\bar{y} = \frac{1}{A} \sum_{(x,y)\in R} y$$



Source: Shapiro & Stockman

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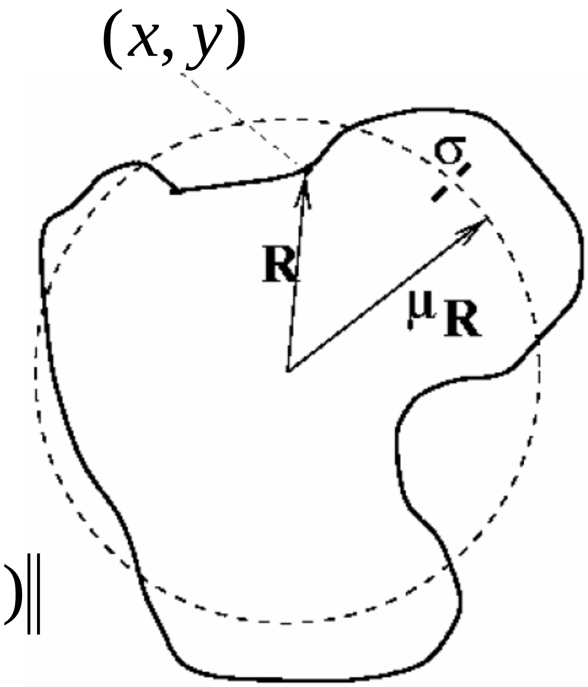
Circularity

- Measure the deviation from a perfect circle
  - *Circularity*: $$C = \frac{\mu_R}{\sigma_R}$$

    where μ_R and σ_R^2 are the mean and variance of the distance from the centroid of the shape to the boundary pixels (x_k, y_k).



  - *Mean radial distance*:

$$\mu_R = \frac{1}{K} \sum_{k=0}^{K-1} \left\| (x_k, y_k) - (\bar{x}, \bar{y}) \right\|$$

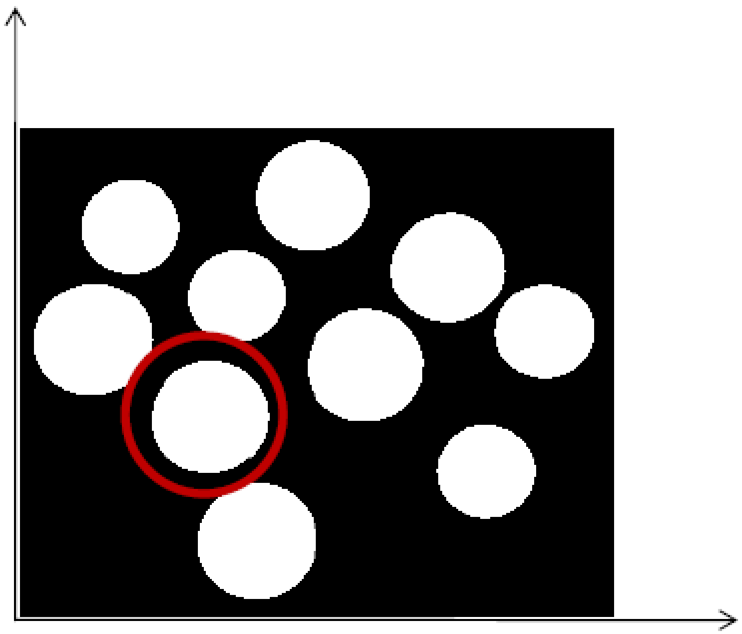
  - *Variance of radial distance*:

$$\sigma_R^2 = \frac{1}{K} \sum_{k=0}^{K-1} \left[\left\| (x_k, y_k) - (\bar{x}, \bar{y}) \right\| - \mu_R \right]^2$$

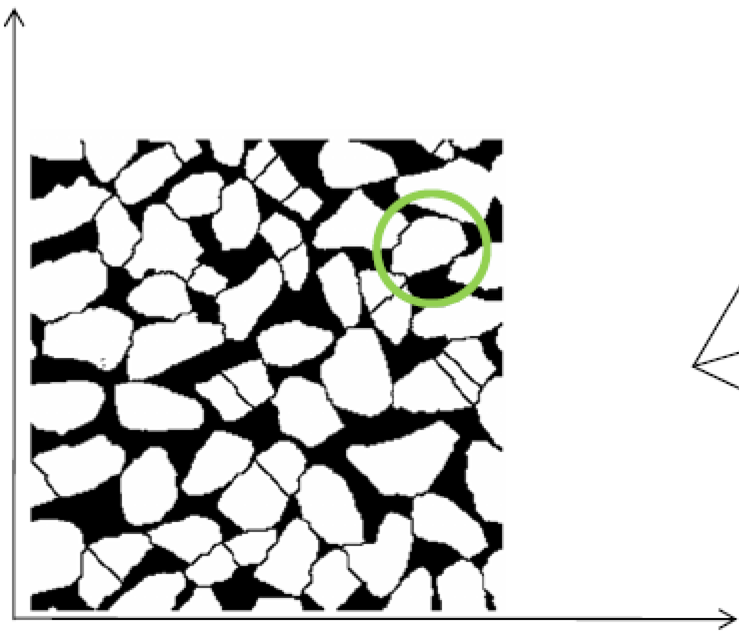
Source: Shapiro & Stockman

Invariant Descriptors

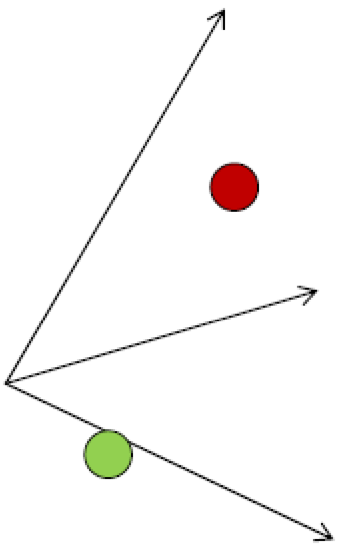
- Often, we want features independent of location, orientation, scale.



$$[a_1, a_2, a_3, \ldots]$$



$$[b_1, b_2, b_3, \ldots]$$



Feature space distance

Central Moments

- S is a subset of pixels (region).
- Central $(j,k)^{\text{th}}$ moment defined as:

$$\mu_{jk} = \sum_{(x,y)\in S} (x-\bar{x})^j (y-\bar{y})^k$$

- Invariant to translation of S.

- **Interpretation:**
  - 0th central moment: *area*
  - 2nd central moment: *variance*
  - 3rd central moment: *skewness*
  - 4th central moment: *kurtosis*

Moment Invariants ("Hu Moments")

- Normalized central moments

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}, \qquad \gamma = \frac{p+q}{2} + 1$$

- From those, a set of *invariant moments* can be defined for object description.

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

- Robust to translation, rotation & scaling, but don't expect wonders (still summary statistics).

Moment Invariants

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right]$$
$$+ (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$

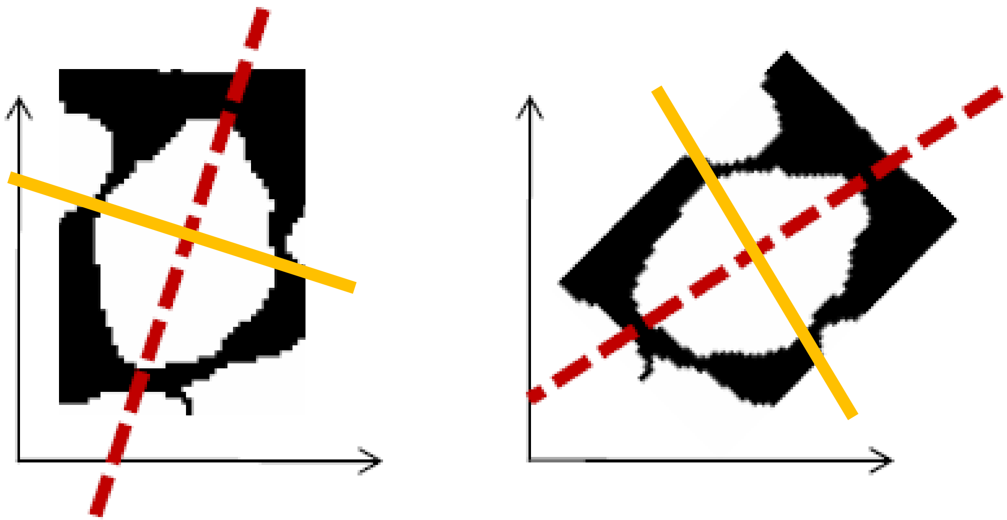
$$\phi_6 = (\eta_{20} - \eta_{02})\left[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$
$$+ 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\phi_7 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})\left[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2\right]$$
$$+ (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03})\left[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2\right]$$

Often better to use $\log_{10}(\phi_i)$ instead of ϕ_i directly…

Axis of Least Second Moment

- **Invariance to orientation?**
  - Need a common alignment



Axis for which the squared distance to 2D object points is minimized (maximized).

  - Compute Eigenvectors of 2[nd] moment matrix (Matlab: eig(A))

$$\begin{bmatrix} \mu_{20} & \mu_{11} \\ \mu_{11} & \mu_{02} \end{bmatrix} = VDV^T = \begin{bmatrix} v_{11} & v_{12} \\ v_{22} & v_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}^T$$

Summary: Binary Image Processing

- Pros
  - Fast to compute, easy to store
  - Simple processing techniques
  - Can be very useful for constrained scenarios
- Cons
  - Hard to get "clean" silhouettes
  - Noise is common in realistic scenarios
  - Can be too coarse a representation
  - Cannot deal with 3D changes

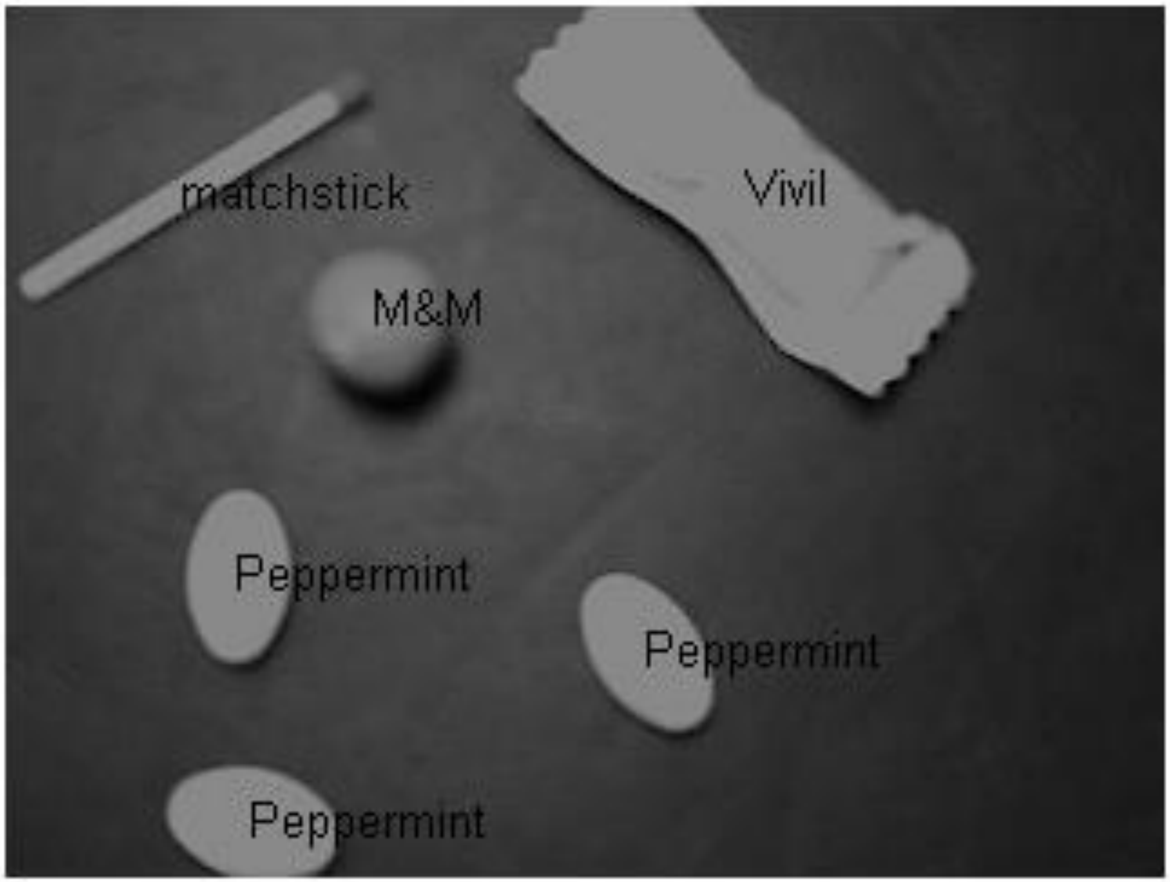
References and Further Reading

- More on morphological operators can be found in
  - R.C. Gonzales, R.E. Woods,
    Digital Image Processing.
    Prentice Hall, 2001

- Online tutorial and Java demos available on
  - http://homepages.inf.ed.ac.uk/rbf/HIPR2/

Questions ?

Demo “Haribo Classification”



You Can Do It At Home…

Accessing a webcam in Matlab:

```
function out = webcam
% uses "Image Acquisition Toolbox„
adaptorName = 'winvideo';
vidFormat = 'I420_320x240';
vidObj1= videoinput(adaptorName, 1, vidFormat);
set(vidObj1, 'ReturnedColorSpace', 'rgb');
set(vidObj1, 'FramesPerTrigger', 1);
out = vidObj1 ;


cam = webcam();
img=getsnapshot(cam);
```

Questions ?