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Computer Vision - Lecture 7

Segmentation as Energy Minimization

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Announcements • Please don't forget to register for the exam! • On the Campus system

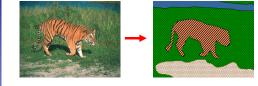
Course Outline

- Image Processing Basics
- Segmentation
 - > Segmentation and Grouping
 - Segmentation as Energy Minimization
- Recognition
 - Global Representations
 - Subspace representations
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
- Motion and Tracking

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Recap: Image Segmentation

· Goal: identify groups of pixels that go together



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Recap: K-Means Clustering

- Basic idea: randomly initialize the k cluster centers, and iterate between the two steps we just saw.
 - 1. Randomly initialize the cluster centers, $c_1,\,...,\,c_K$
 - 2. Given cluster centers, determine points in each cluster
 - For each point p, find the closest c_i. Put p into cluster i
 - 3. Given points in each cluster, solve for \boldsymbol{c}_{i}
 - Set c_i to be the mean of points in cluster i 4. If c_i have changed, repeat Step 2

Properties

- Will always converge to some solution
- > Can be a "local minimum"
 - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points p in cluster } i} ||p - c_i||^2$$

Slide credit: Steve Seitz

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Recap: Expectation Maximization (EM)

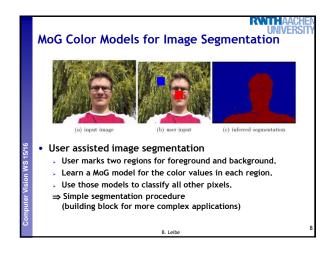
- Goa
 - \succ Find blob parameters θ that maximize the likelihood function:

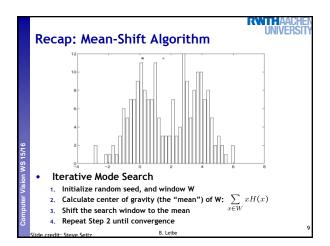
$$p(data|\theta) = \prod_{n=1}^{N} p(\mathbf{x}_n|\theta)$$

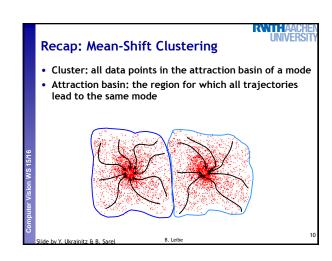
- Approach:
 - 1. E-step: given current guess of blobs, compute ownership of each point
 - 2. M-step: given ownership probabilities, update blobs to maximize likelihood function
 - 3. Repeat until convergence

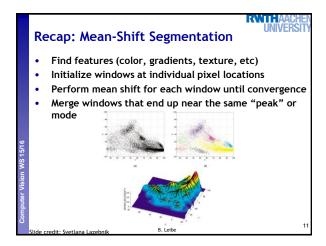
credit: Steve Seitz B

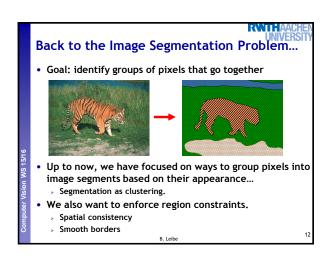
Recap: EM Algorithm • Expectation-Maximization (EM) Algorithm • E-Step: softly assign samples to mixture components $\gamma_j(\mathbf{x}_n) \leftarrow \frac{\pi_j \mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)}{\sum_{k=1}^K \pi_k \mathcal{N}(\mathbf{x}_n | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)} \quad \forall j=1,\dots,K, \ n=1,\dots,N$ • M-Step: re-estimate the parameters (separately for each mixture component) based on the soft assignments $\hat{N}_j \leftarrow \sum_{n=1}^N \gamma_j(\mathbf{x}_n) = \text{soft number of samples labeled } j$ $\hat{\pi}_j^{\text{new}} \leftarrow \frac{\hat{N}_j}{N}$ $\hat{\mu}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^N \gamma_j(\mathbf{x}_n) \mathbf{x}_n$ $\hat{\Sigma}_j^{\text{new}} \leftarrow \frac{1}{\hat{N}_j} \sum_{n=1}^N \gamma_j(\mathbf{x}_n) (\mathbf{x}_n - \hat{\mu}_j^{\text{new}}) (\mathbf{x}_n - \hat{\mu}_j^{\text{new}})^{\text{T}}$ but a adapted from Bernt Schiele





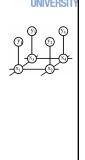




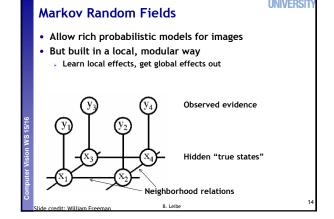


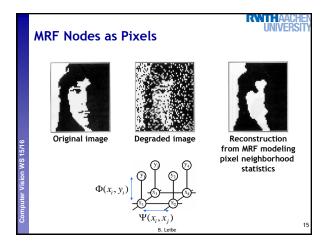
Topics of This Lecture

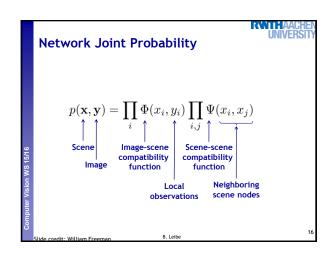
- · Segmentation as Energy Minimization
 - Markov Random Fields
 - > Energy formulation
- Graph cuts for image segmentation
 - Basic idea
 - > s-t Mincut algorithm
 - > Extension to non-binary case
- Applications
 - > Interactive segmentation



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Energy Formulation

Joint probability

$$p(\mathbf{x}, \mathbf{y}) = \prod_{i} \Phi(x_i, y_i) \prod_{i,j} \Psi(x_i, x_j)$$

 Maximizing the joint probability is the same as minimizing the negative log

$$\begin{aligned} -\log p(\mathbf{x}, \mathbf{y}) &= & -\sum_{i} \log \Phi(x_i, y_i) - \sum_{i,j} \log \Psi(x_i, x_j) \\ E(\mathbf{x}, \mathbf{y}) &= & \sum_{i} \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j) \end{aligned}$$

- This is similar to free-energy problems in statistical mechanics (spin glass theory). We therefore draw the analogy and call E an energy function.
- ϕ and ψ are called potentials.

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Energy Formulation

· Energy function

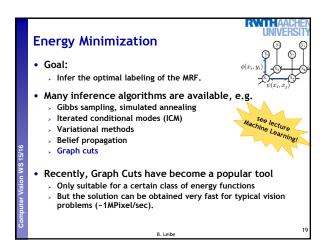
$$E(\mathbf{x}, \mathbf{y}) = \sum_{i} \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j)$$

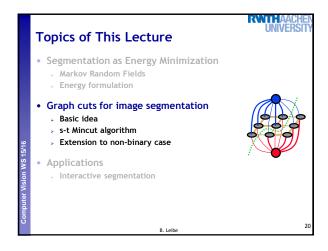
Single-node potentials

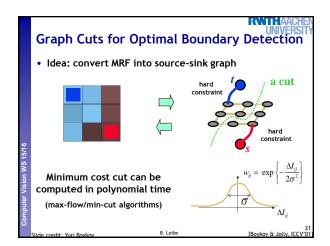
Pairwise potentials

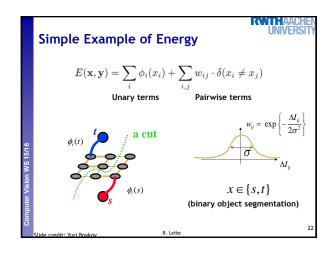
- Single-node potentials ϕ ("unary potentials")
 - \succ Encode local information about the given pixel/patch
 - How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?
- Pairwise potentials ψ
 - > Encode neighborhood information
 - How different is a pixel/patch's label from that of its neighbor? (e.g. based on intensity/color/texture difference, edges)

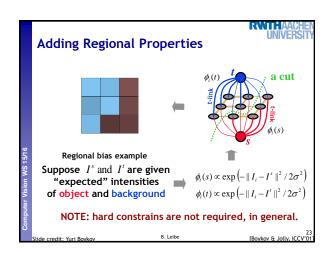
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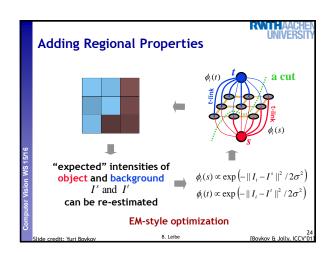


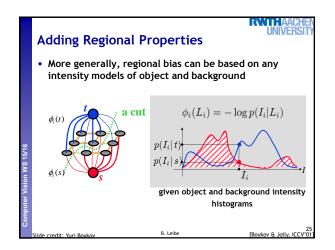


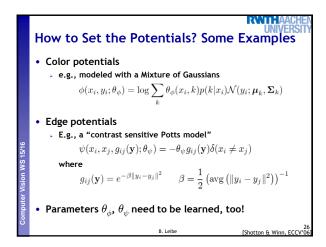


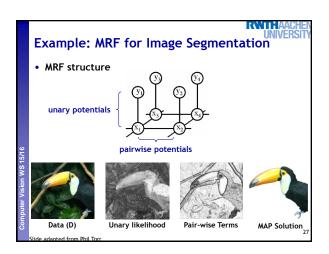


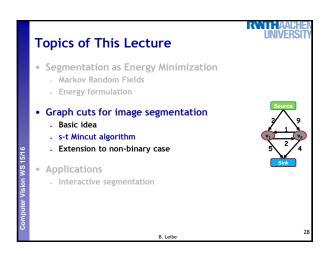


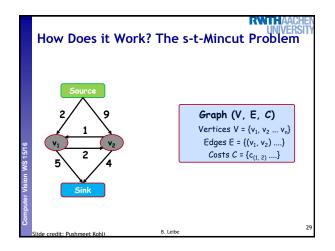


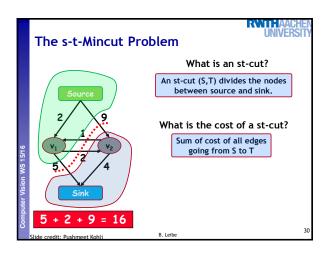


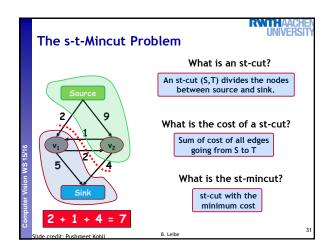


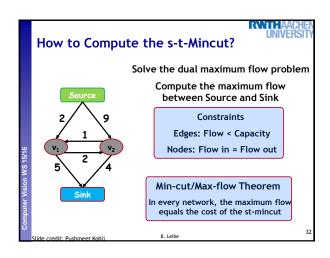


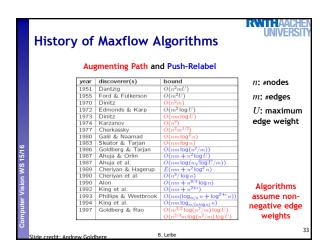


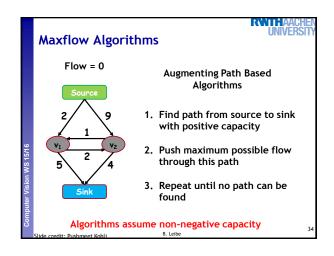


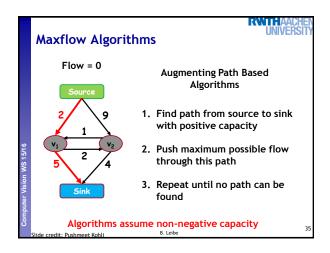


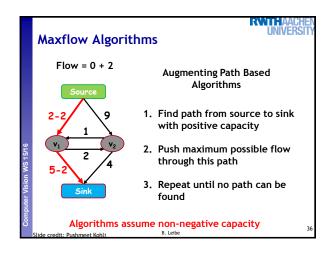


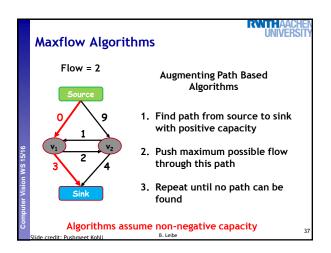


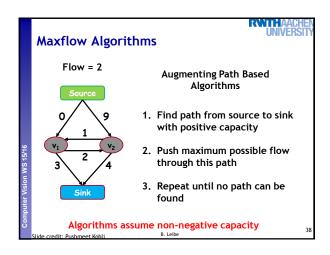


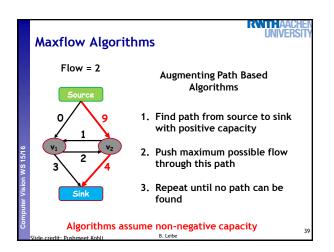


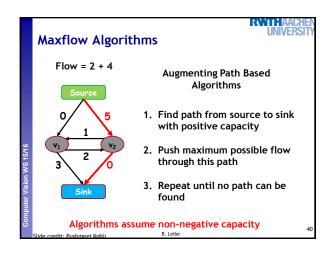


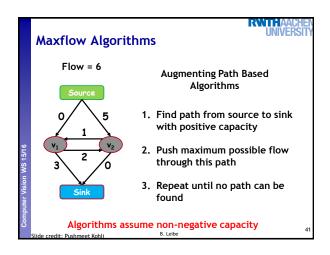


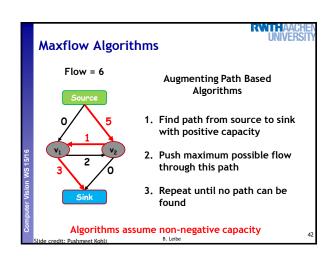


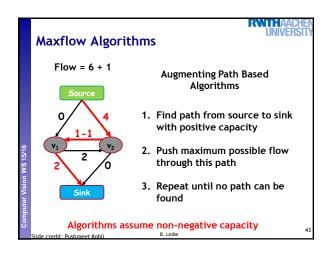


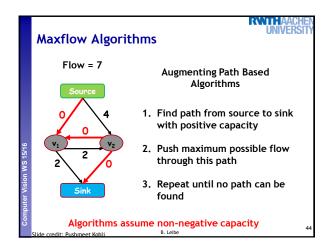


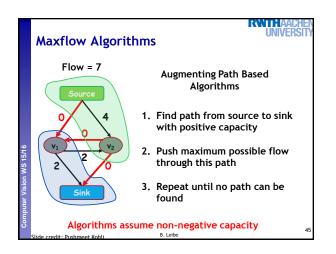


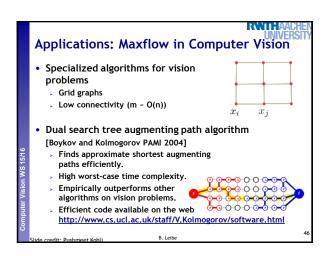


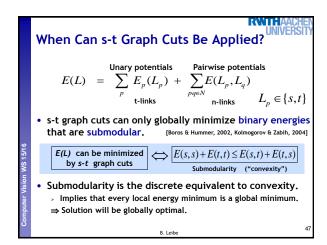


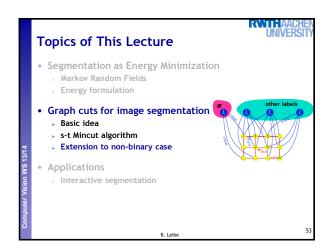




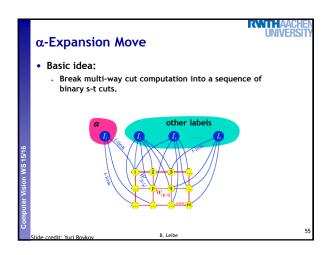


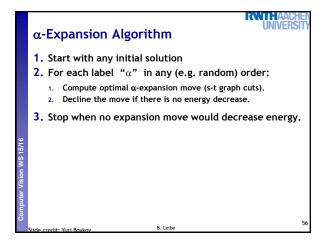


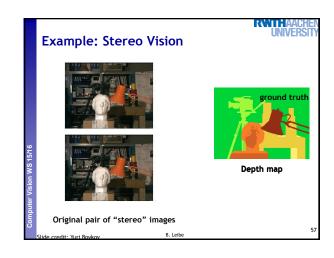


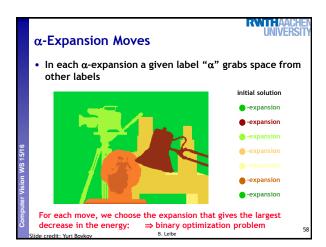


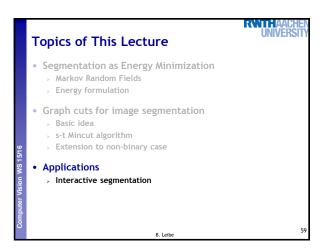
Dealing with Non-Binary Cases • Limitation to binary energies is often a nuisance. ⇒ E.g. binary segmentation only... • We would like to solve also multi-label problems. • The bad news: Problem is NP-hard with 3 or more labels! • There exist some approximation algorithms which extend graph cuts to the multi-label case: • α-Expansion • αβ-Swap • They are no longer guaranteed to return the globally optimal result. • But α-Expansion has a guaranteed approximation quality (2-approx) and converges in a few iterations.

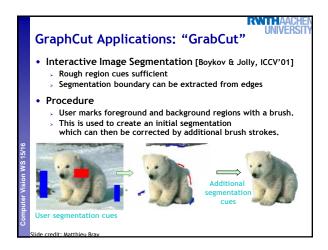


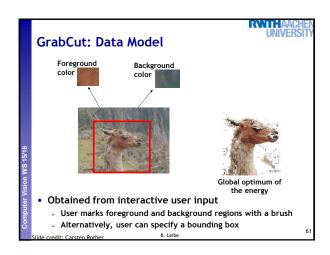


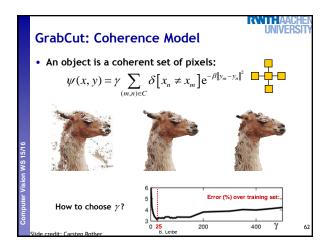


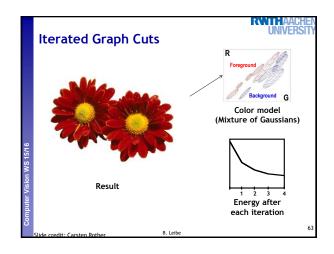


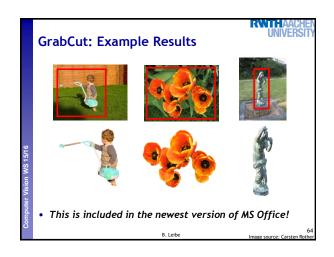


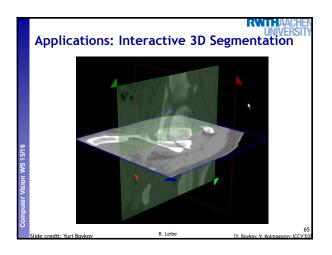














Summary: Graph Cuts Segmentation

Pros

- > Powerful technique, based on probabilistic model (MRF).
- > Applicable for a wide range of problems.
- > Very efficient algorithms available for vision problems.
- > Becoming a de-facto standard for many segmentation tasks.

Cons/Issues

- > Graph cuts can only solve a limited class of models
 - Submodular energy functions
 - Can capture only part of the expressiveness of MRFs
- > Only approximate algorithms available for multi-label case

References and Further Reading

 A gentle introduction to Graph Cuts can be found in the following paper:

- Y. Boykov, O. Veksler, <u>Graph Cuts in Vision and Graphics: Theories and Applications</u>. In *Handbook of Mathematical Models in Computer Vision*, edited by N. Paragios, Y. Chen and O. Faugeras, Springer, 2006.
- Read how the interactive segmentation is realized in MS Office 2010
 - C. Rother, V. Kolmogorov, Y. Boykov, A. Blake, <u>Interactive</u>
 <u>Foreground Extraction using Graph Cut</u>, Microsoft Research Tech
 Report MSR-TR-2011-46, March 2011
- Try the GraphCut implementation at http://www.cs.ucl.ac.uk/staff/V.Kolmogorov/software.html

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