Computer Vision – Lecture 15

Camera Calibration & 3D Reconstruction

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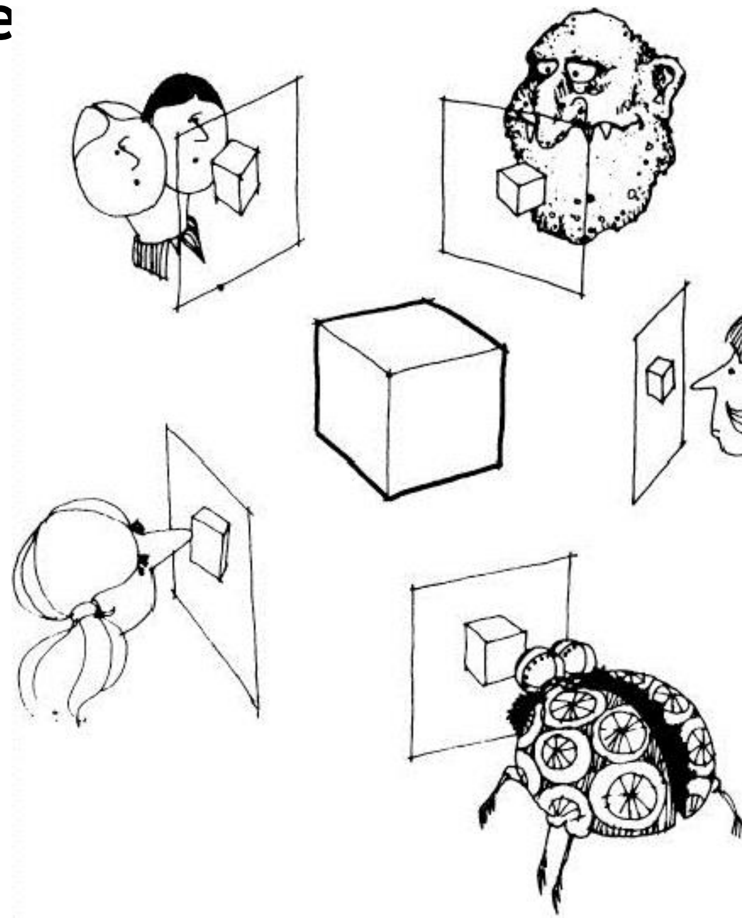
Course Outline

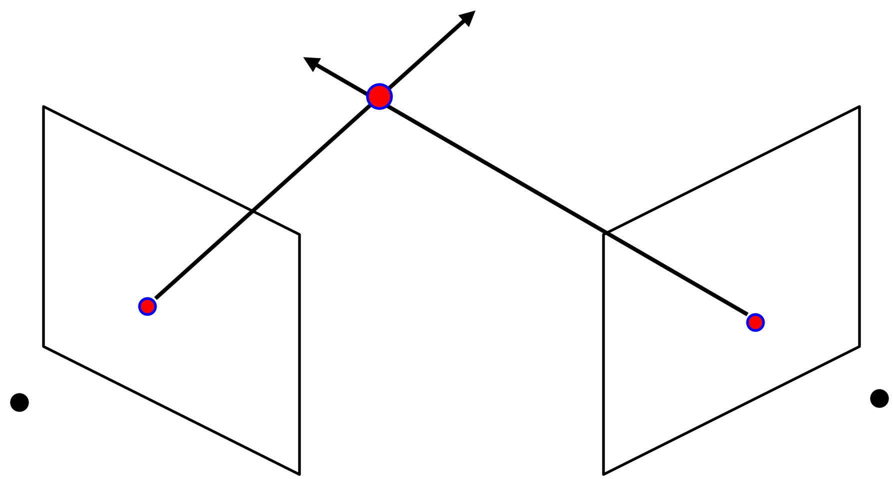
- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
  - Epipolar Geometry and Stereo Basics
  - Camera calibration & Uncalibrated Reconstruction
  - Structure-from-Motion
- Motion and Tracking

Recap: What Is Stereo Vision?

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape



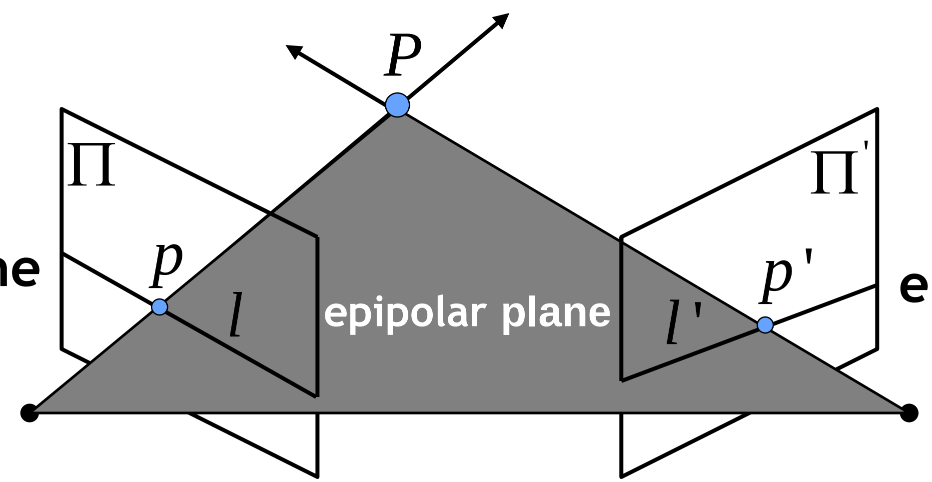
Recap: Depth with Stereo - Basic Idea



- Basic Principle: Triangulation
  - Gives reconstruction as intersection of two rays
  - Requires
    - Camera pose (calibration)
    - Point correspondence

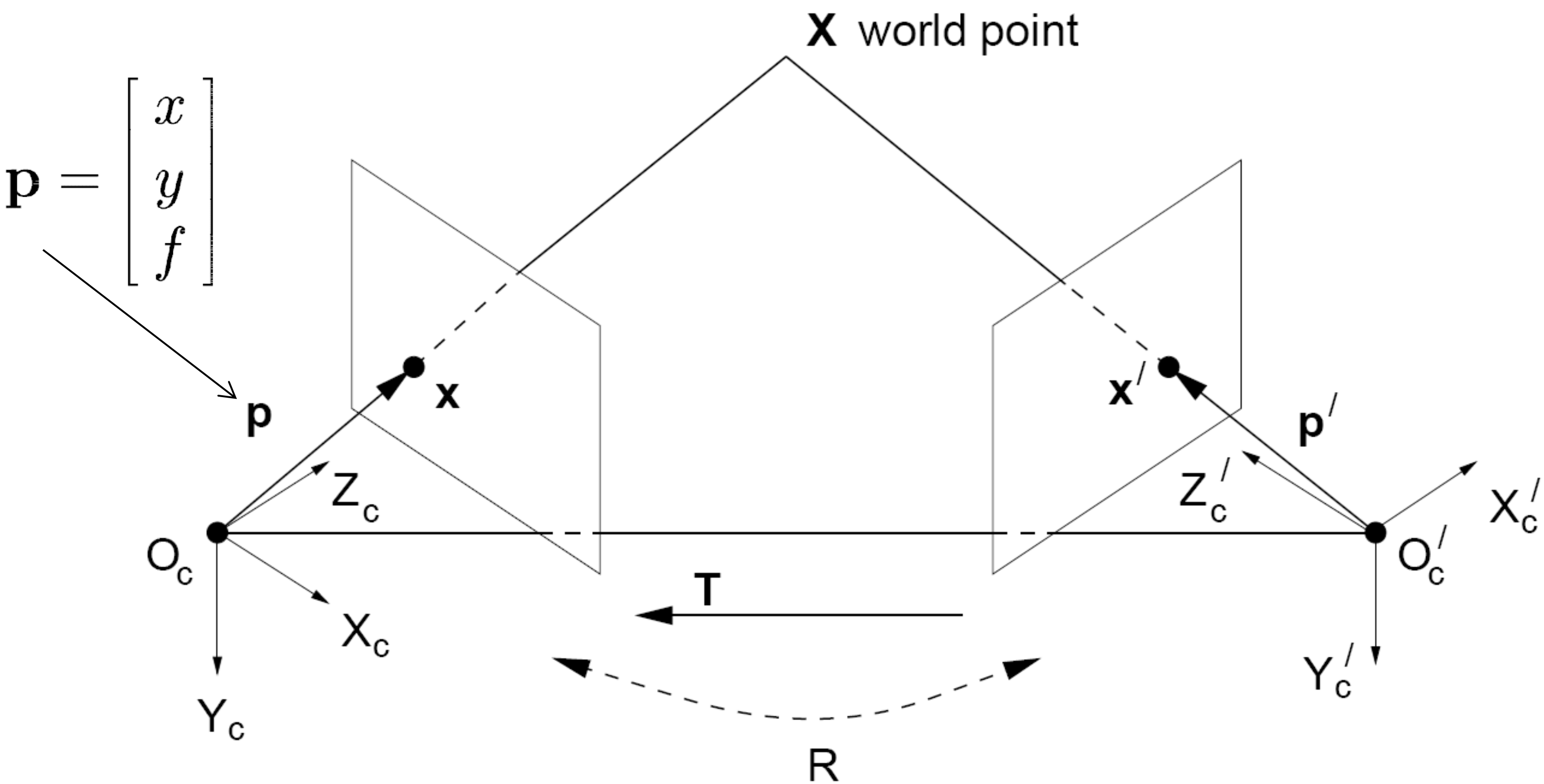
Recap: Epipolar Geometry

- Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.



- Epipolar constraint:
  - Correspondence for point p in Π must lie on the epipolar line l' in Π' (and vice versa).
  - Reduces correspondence problem to 1D search along conjugate epipolar lines.

Recap: Stereo Geometry With Calibrated Cameras



- Camera-centered coordinate systems are related by known rotation R and translation T:

$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T}$$

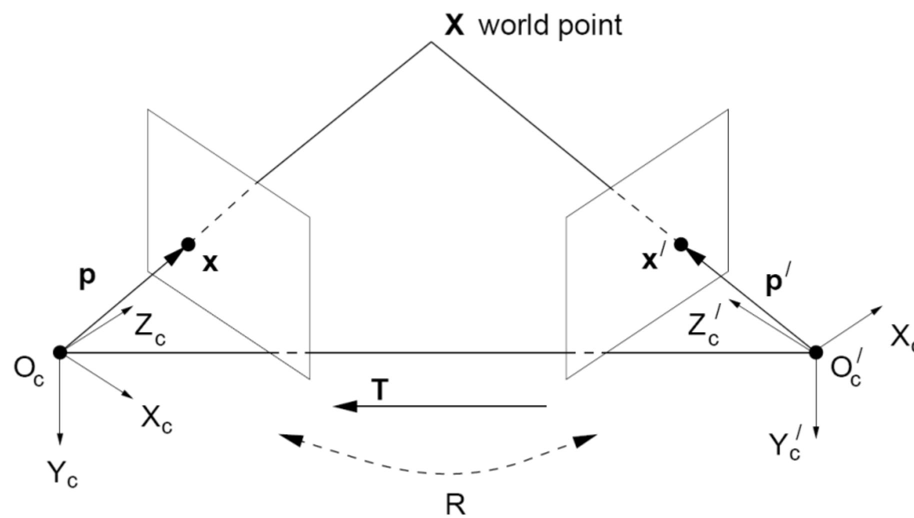
Recap: Essential Matrix

$$\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X}) = 0$$

$$\mathbf{X}' \cdot (\mathbf{T}_x \ \mathbf{R}\mathbf{X}) = 0$$

Let $\mathbf{E} = \mathbf{T}_x\mathbf{R}$

$$\mathbf{X}'^T \mathbf{E}\mathbf{X} = 0$$



- This holds for the rays p and p’ that are parallel to the camera-centered position vectors X and X’, so we have: $\boxed{\mathbf{p}'^T \mathbf{E}\mathbf{p} = 0}$
- $\mathbf{E}$ is called the essential matrix, which relates corresponding image points [Longuet-Higgins 1981]

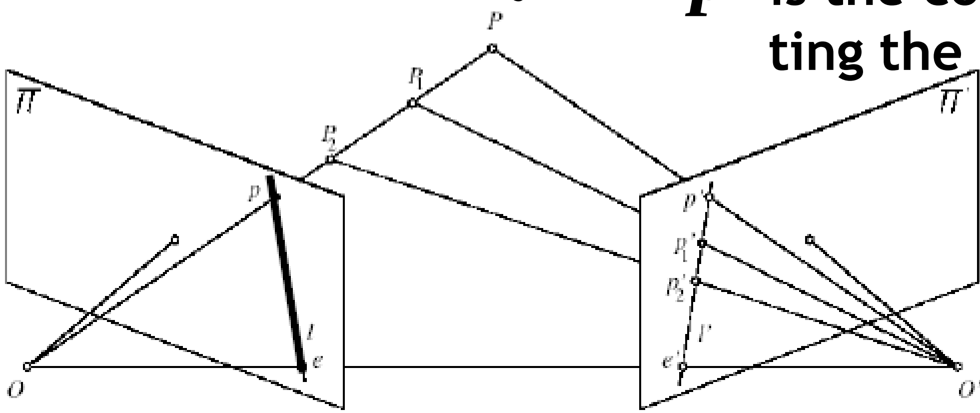
Recap: Essential Matrix and Epipolar Lines

$$\boxed{\mathbf{p'}^{\mathrm{T}}\mathbf{E}\mathbf{p} = 0}$$

Epipolar constraint: if we observe point p in one image, then its position p' in second image must satisfy this equation.

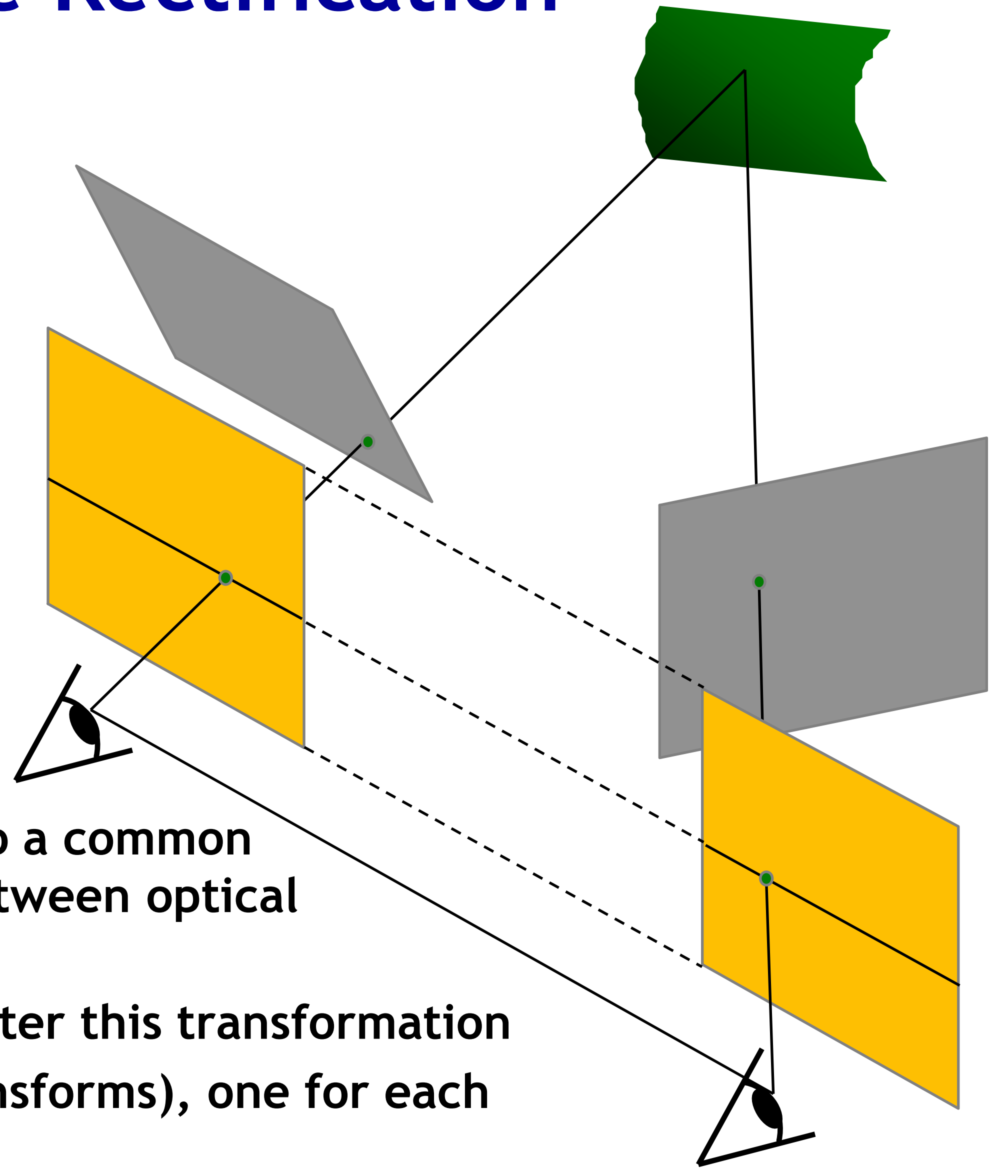
$\boldsymbol{l}' = \boldsymbol{E}\boldsymbol{p}$ is the coordinate vector representing the epipolar line for point p

(i.e., the line is given by: $l'^{\top}\mathbf{x} = 0$)



$\boldsymbol{l} = \boldsymbol{E}^{T}\boldsymbol{p}'$ is the coordinate vector representing the epipolar line for point p'

Recap: Stereo Image Rectification



- In practice, it is convenient if image scanlines are the epipolar lines.

- Algorithm
  - Reproject image planes onto a common plane parallel to the line between optical centers
  - Pixel motion is horizontal after this transformation
  - Two homographies (3x3 transforms), one for each input image reprojection

Slide adapted from Li Zhang

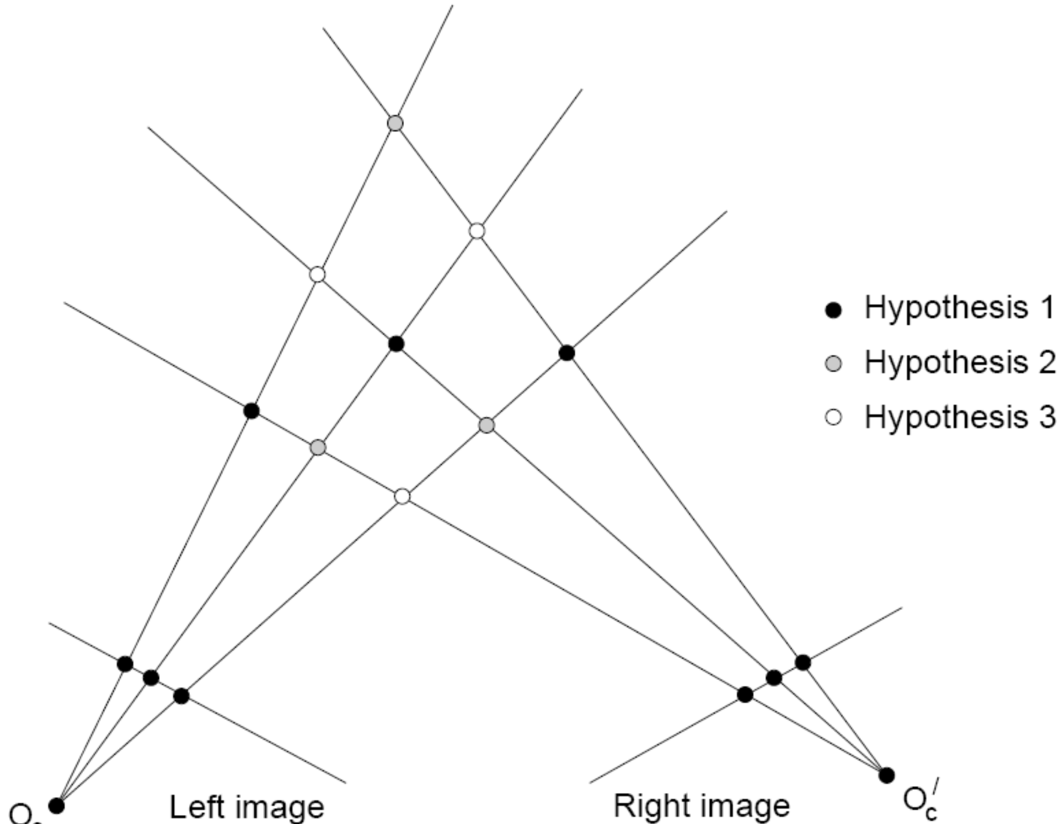
C. Loop & Z. Zhang, [Computing Rectifying Homographies for Stereo Vision](). CVPR'99

Recap: Stereo Reconstruction

- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

Slide credit: Kristen Grauman

Correspondence Problem

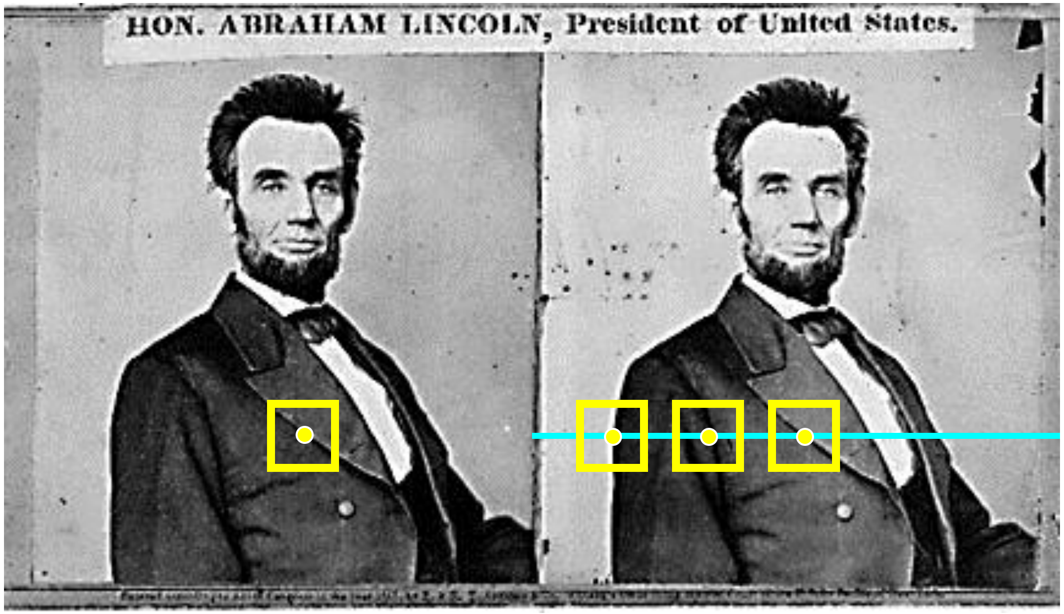


Multiple match hypotheses satisfy epipolar constraint, but which is correct?

Slide credit: Kristen Grauman

Figure from Gee & Cipolla 1999

Dense Correspondence Search



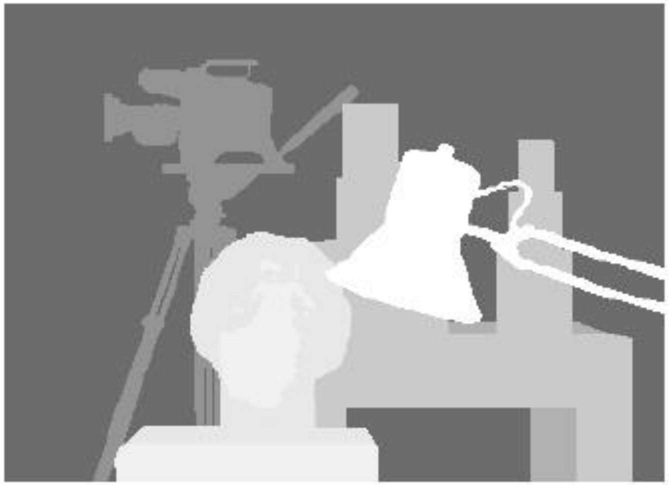
- For each pixel in the first image
  - Find corresponding epipolar line in the right image
  - Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
  - Triangulate the matches to get depth information
- This is easiest when epipolar lines are scanlines
  - $\Rightarrow$ Rectify images first

adapted from Svetlana Lazebnik, Li Zhang

Example: Window Search

- **Data from University of Tsukuba**

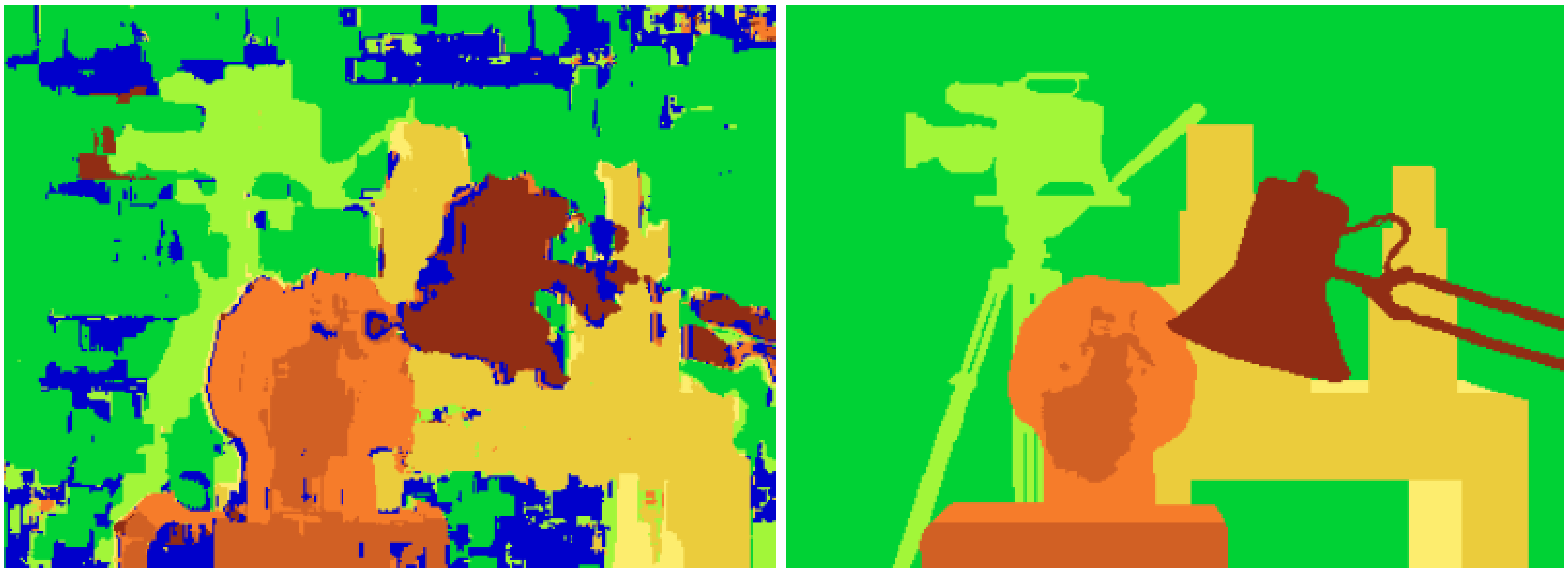
Scene



Ground truth

Example: Window Search

- **Data from University of Tsukuba**



Window-based matching
(best window size)

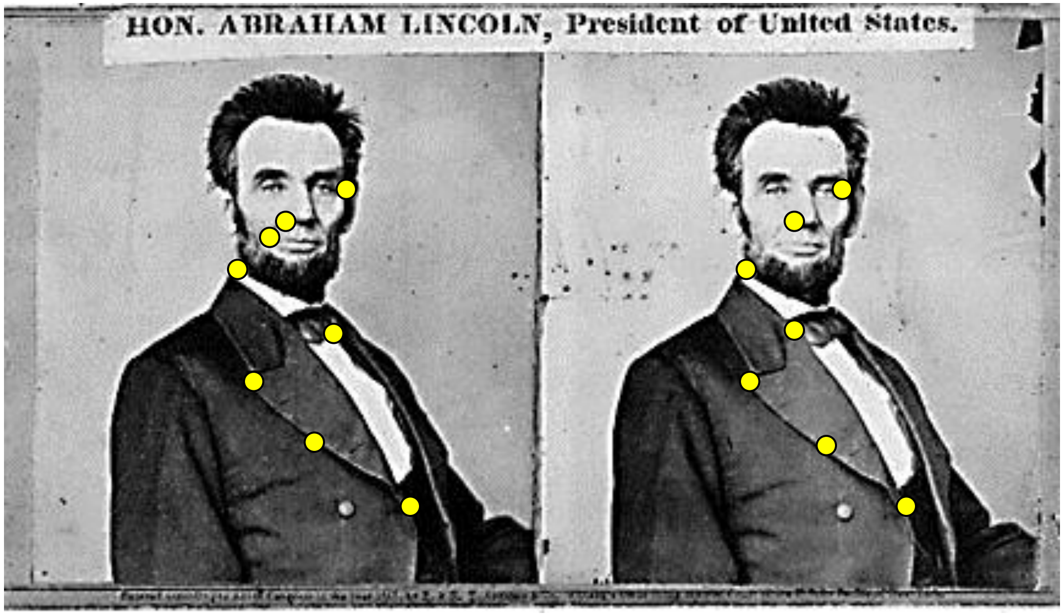
Ground truth

Effect of Window Size

$W = 3$ $W = 20$

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Alternative: Sparse Correspondence Search



- **Idea:**
  - Restrict search to sparse set of detected features
  - Rather than pixel values (or lists of pixel values) use *feature descriptor* and an associated *feature distance*
  - Still narrow search further by epipolar geometry

What would make good features?

Dense vs. Sparse

- Sparse
  - Efficiency
  - Can have more reliable feature matches, less sensitive to illumination than raw pixels
  - But…
    - Have to know enough to pick good features
    - Sparse information
- Dense
  - Simple process
  - More depth estimates, can be useful for surface reconstruction
  - But…
    - Breaks down in textureless regions anyway
    - Raw pixel distances can be brittle
    - Not good with very different viewpoints

Slide credit: Kristen Grauman

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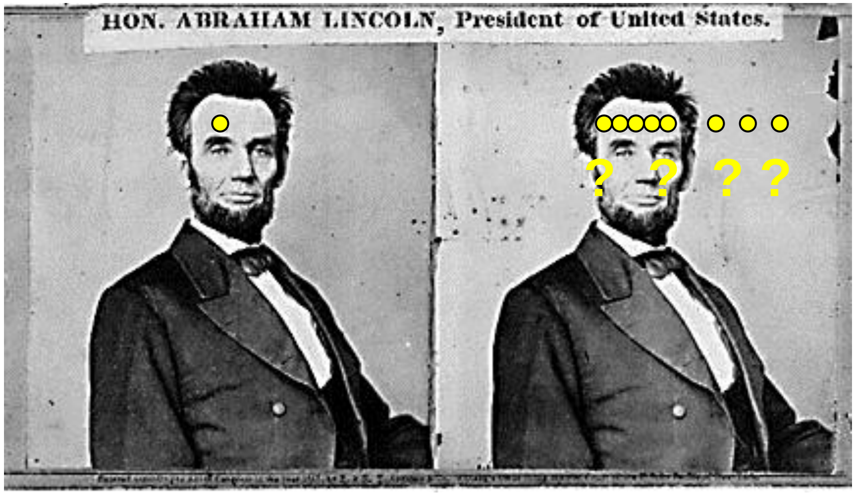
Difficulties in Similarity Constraint



Untextured surfaces

Occlusions

Possible Sources of Error?

- Low-contrast / textureless image regions
- Occlusions
- Camera calibration errors
- Violations of *brightness constancy* (e.g., specular reflections)
- Large motions

Summary: Stereo Reconstruction

- Main Steps
  - Calibrate cameras
  - Rectify images
  - Compute disparity
  - Estimate depth

Left

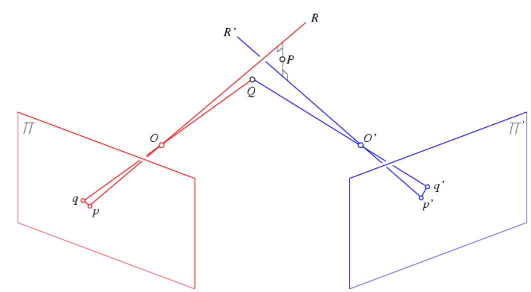
Right

- So far, we have only considered calibrated cameras…

Left

Right

- Today
  - Uncalibrated cameras
  - Camera parameters
  - Revisiting epipolar geometry
  - Robust fitting



Recap: A General Point

- Equations of the form

$$\mathrm{Ax} = 0$$

- How do we solve them? (always!)
  - Apply SVD

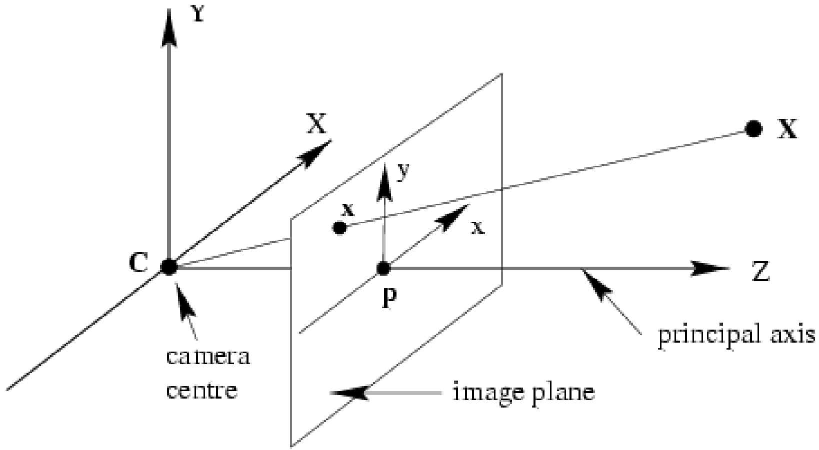
$$\boldsymbol{A} \overset{\text{SVD}}{=} \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^T = \boldsymbol{U} \underbrace{\begin{bmatrix} d_{11} & & \\ & \ddots & \\ & & d_{NN} \end{bmatrix}}_{\text{Singular values}} \underbrace{\begin{bmatrix} v_{11} & \cdots & \boxed{v_{1N}} \\ \vdots & \ddots & \vdots \\ v_{N1} & \cdots & v_{NN} \end{bmatrix}^T}_{\text{Singular vectors}}$$

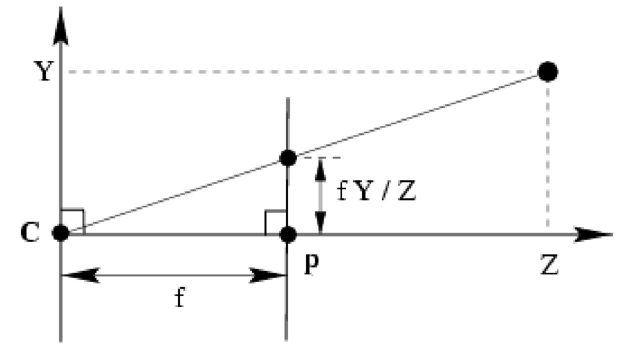
  - Singular values of A = square roots of the eigenvalues of A^TA.
  - The solution of Ax=0 is the *nullspace* vector of A.
  - This corresponds to the *smallest singular vector* of A.

Topics of This Lecture

- Camera Calibration
  - Camera parameters
  - Calibration procedure
- Revisiting Epipolar Geometry
  - Triangulation
  - Calibrated case: Essential matrix
  - Uncalibrated case: Fundamental matrix
  - Weak calibration
  - Epipolar Transfer
- Active Stereo
  - Laser scanning
  - Kinect sensor

Recall: Pinhole Camera Model





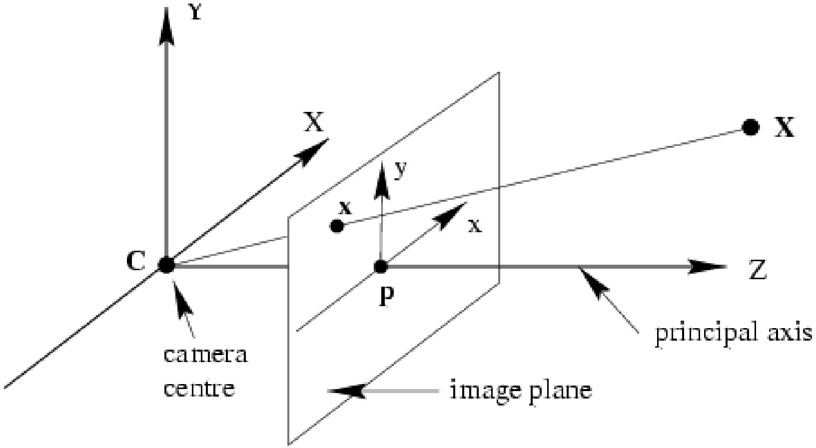
$$(X, Y, Z) \mapsto (f\,X/Z, f\,Y/Z)$$

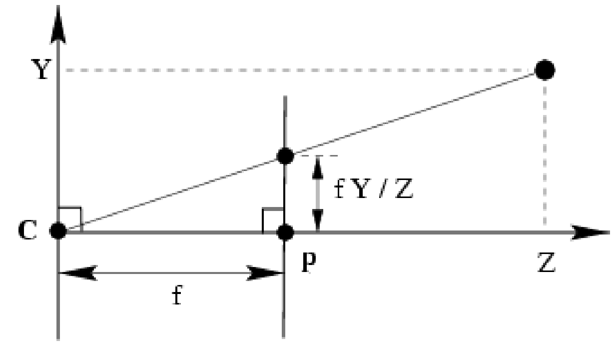
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{PX}$$

Images from Hartley & Zisserman

Pinhole Camera Model

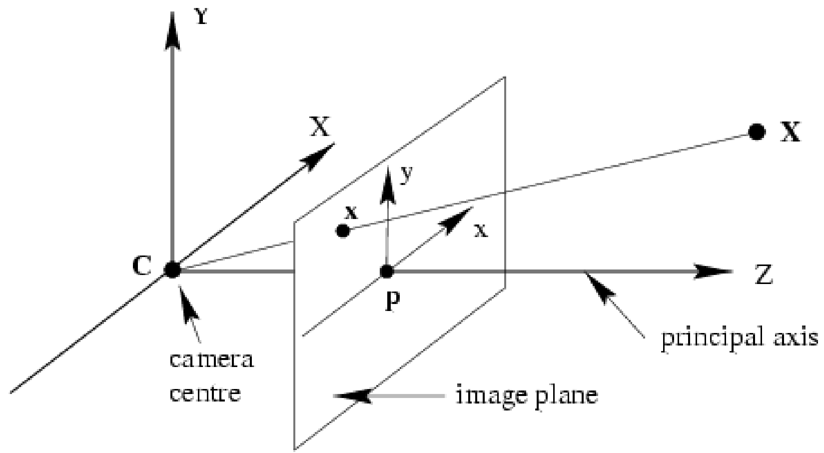




$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & \\ & f & \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\mathrm{x} = \mathrm{PX} \qquad \mathrm{P} = \mathrm{diag}(f, f, 1)[\mathrm{I} \,|\, 0]$$

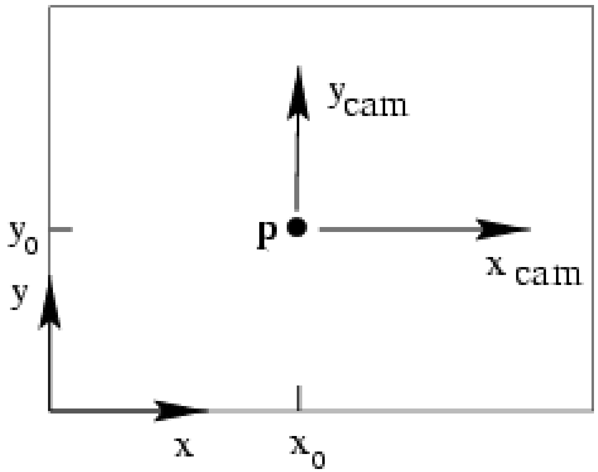
Camera Coordinate System



- *Principal axis*:
  - Line from the camera center perpendicular to the image plane
- *Normalized (camera) coordinate system*:
  - Camera center is at the origin and the principal axis is the z-axis
- *Principal point* (p):
  - Point where principal axis intersects the image plane (origin of normalized coordinate system)

Image from Hartley & Zisserman

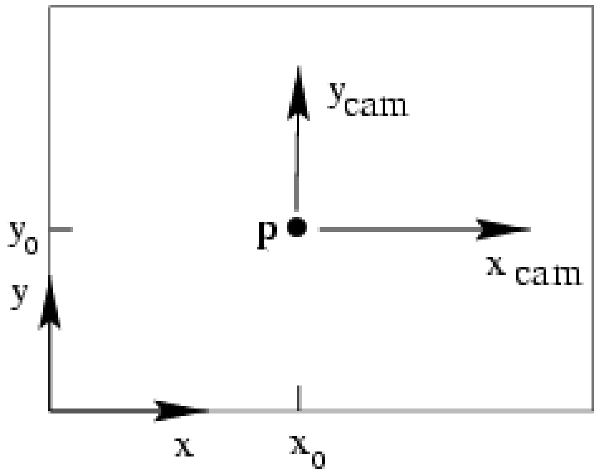
Principal Point Offset



principal point: (p_x, p_y)

- **Camera coordinate system: origin at the principal point**
- **Image coordinate system: origin is in the corner**

Principal Point Offset

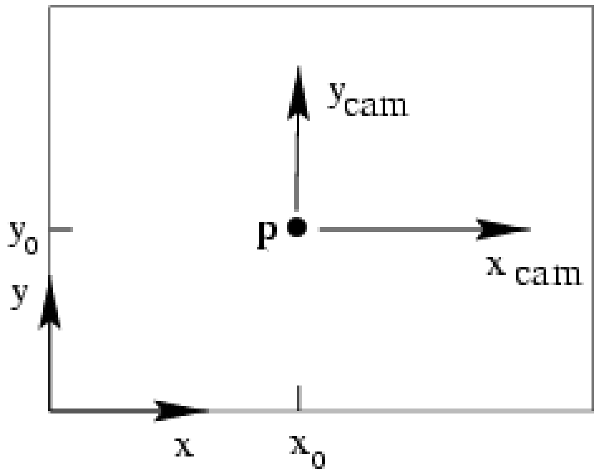


principal point: (p_x, p_y)

$$(X, Y, Z) \mapsto (f\,X/Z + p_x,\ f\,Y/Z + p_y)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} f\,X + Z\,p_x \\ f\,Y + Z\,p_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Principal Point Offset



principal point: (p_x, p_y)

$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_x \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & 0 \\ & 1 & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$K = \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix}$$ **calibration matrix** $$\mathrm{P} = \mathrm{K}[\mathrm{I} \mid 0]$$

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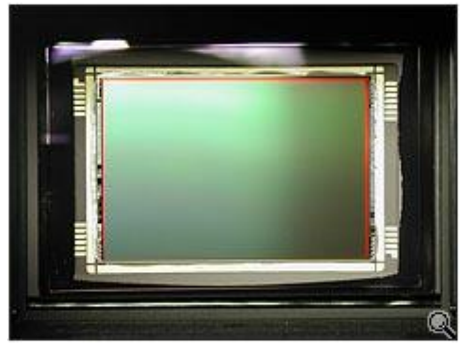
Pixel Coordinates: Non-Square Pixels



Pixel size: $\frac{1}{m_x} \times \frac{1}{m_y}$

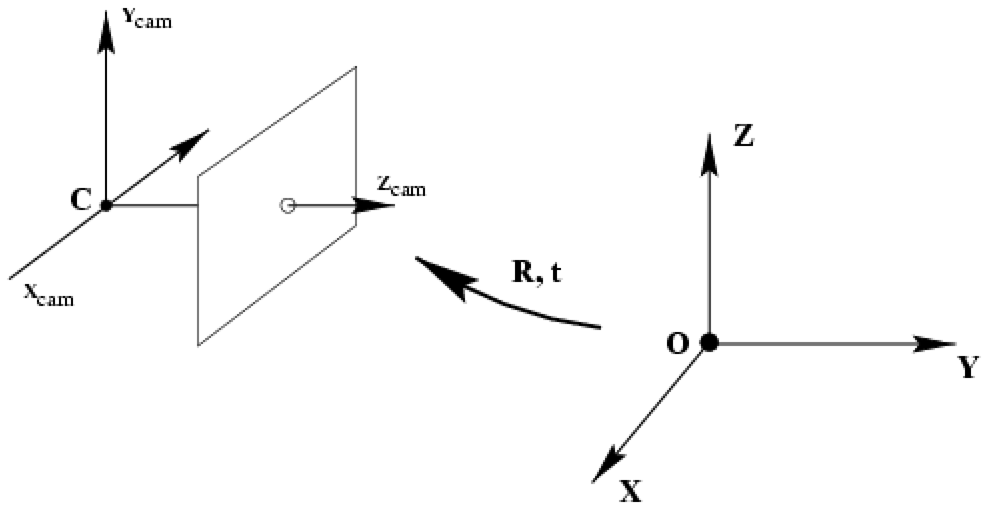
m_x **pixels per meter in horizontal direction,**
m_y **pixels per meter in vertical direction**

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

pixels/m **m** **pixels**

Slide credit: Svetlana Lazebnik

Camera Rotation and Translation



- In general, the camera coordinate frame will be related to the world coordinate frame by a rotation and a translation

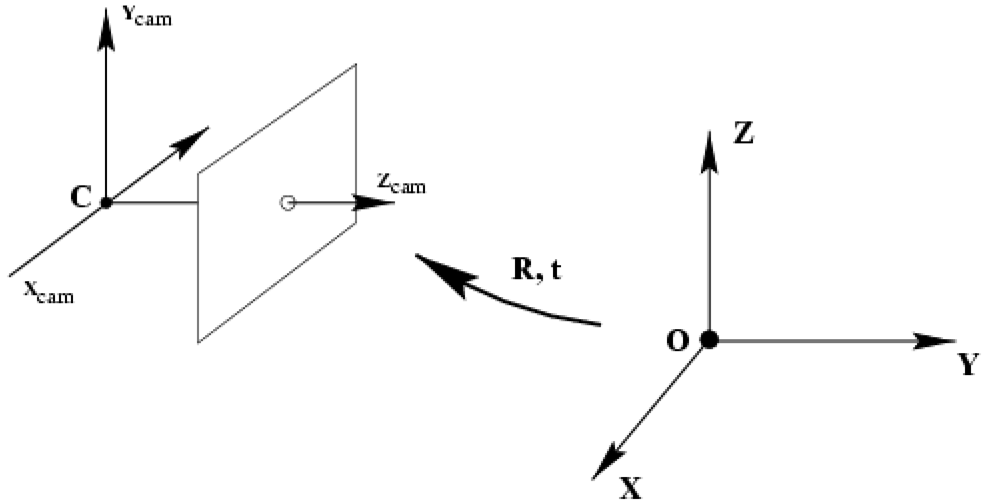
$$\tilde{X}_{cam} = R\left(\tilde{X} - \tilde{C}\right)$$

coords. of point in camera frame

coords. of a point in world frame (nonhomogeneous)

coords. of camera center in world frame

Camera Rotation and Translation



In non-homogeneous coordinates:

$$\tilde{X}_{cam} = R(\tilde{X} - \tilde{C})$$

$$X_{cam} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \tilde{X} \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

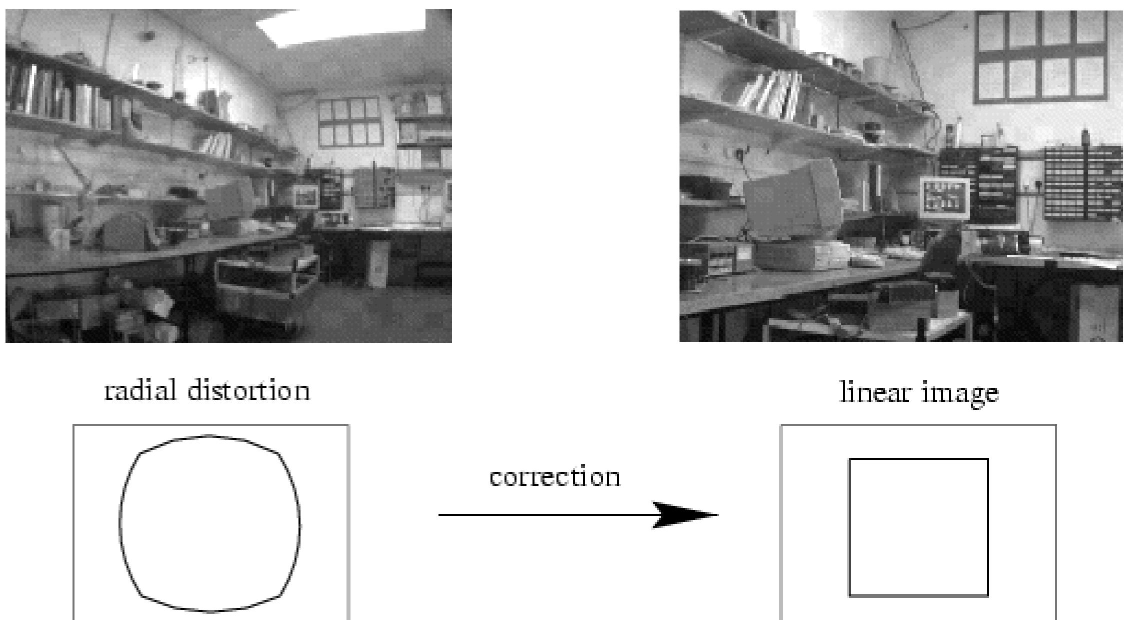
$$x = K[I\,|\,0]X_{cam} = K\left[R\,|\,-R\tilde{C}\right]X \qquad P = K[R\,|\,t], \qquad t = -R\tilde{C}$$

Note: $\mathbf{C}$ is the null space of the camera projection matrix ($\mathbf{PC}=0$)

Summary: Camera Parameters

- Intrinsic parameters
  - Principal point coordinates
  - Focal length
  - Pixel magnification factors
  - *Skew (non-rectangular pixels)*
  - *Radial distortion*

$$K = \begin{bmatrix} m_x & & \\ & m_y & \\ & & 1 \end{bmatrix} \begin{bmatrix} f & s & p_x \\ & f & p_y \\ & & 1 \end{bmatrix} = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$



Summary: Camera Parameters

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- **Extrinsic parameters**
  - Rotation R
  - Translation t
    (both relative to world coordinate system)

- **Camera projection matrix**

$$\mathrm{P} = \mathrm{K}\left[\mathrm{R} \mid \mathrm{t}\right] = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

How many degrees of freedom does P have?

Camera Parameters: Degrees of Freedom

- **Intrinsic parameters** — **DoF**
  - Principal point coordinates — 2
  - Focal length — 1
  - Pixel magnification factors — 1
  - *Skew (non-rectangular pixels)* — 1
  - *Radial distortion*

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

- **Extrinsic parameters**
  - Rotation R — 3
  - Translation t — 3
    (both relative to world coordinate system)

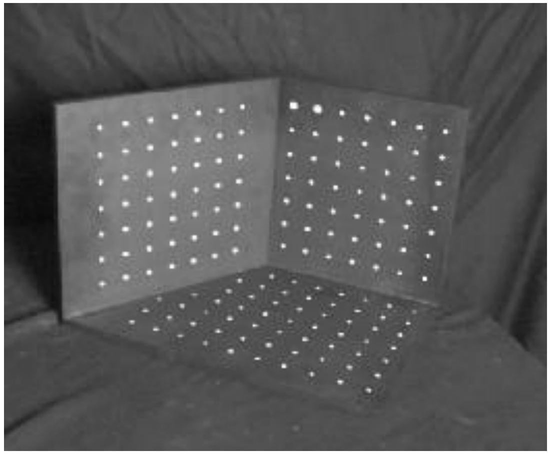
- **Camera projection matrix**

$$\mathrm{P} = \mathrm{K}\left[\mathrm{R} \mid \mathrm{t}\right]$$

⇒ General pinhole camera: 9 DoF
⇒ CCD Camera with square pixels: 10 DoF
⇒ General camera: 11 DoF

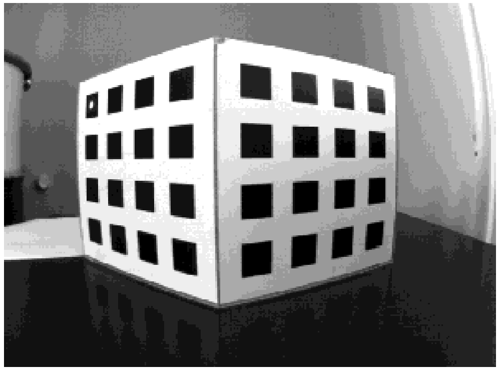
Calibrating a Camera

- Compute intrinsic and extrinsic parameters using observed camera data.



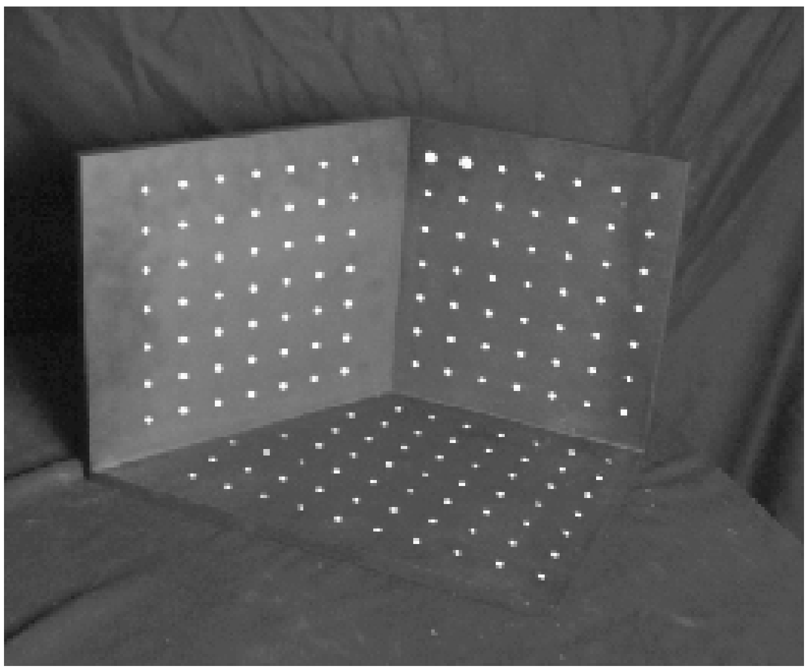
Main idea

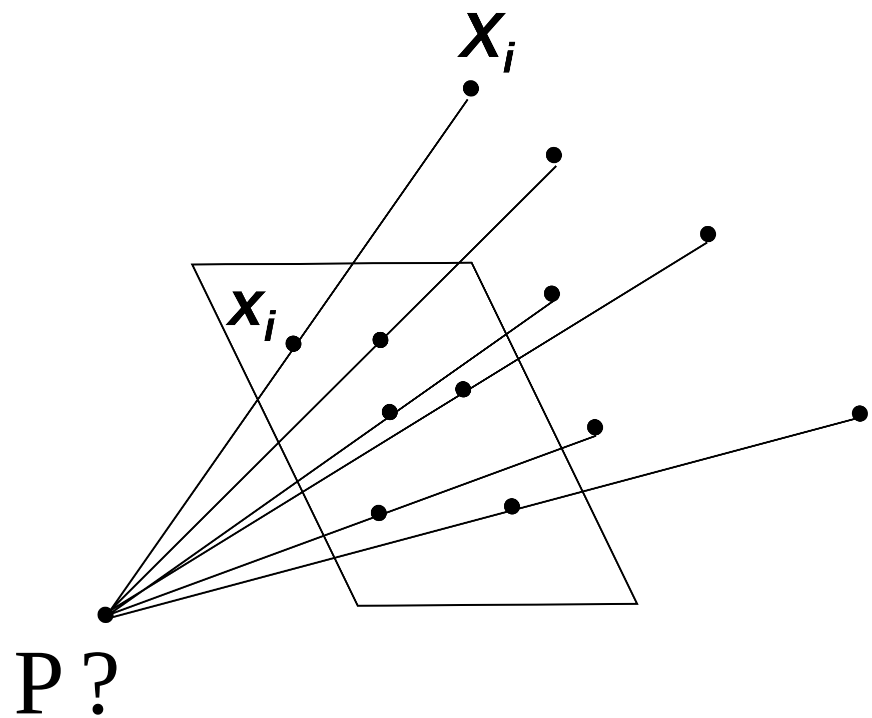
- Place "calibration object" with known geometry in the scene
- Get correspondences
- Solve for mapping from scene to image: estimate $P = P_{int} P_{ext}$



Camera Calibration

- Given n points with known 3D coordinates X_i and known image projections x_i, estimate the camera parameters

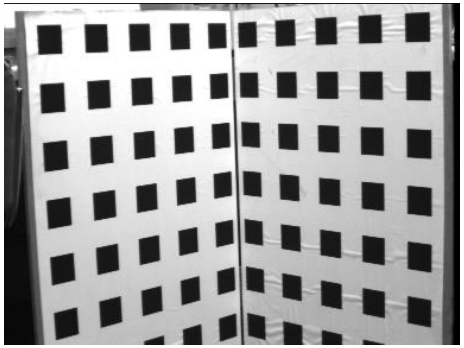




Camera Calibration: Obtaining the Points

- For best results, it is important that the calibration points are measured with *subpixel accuracy*.
- How this can be done depends on the exact pattern.

- Algorithm for checkerboard pattern
  1. Perform Canny edge detection.
  2. Fit straight lines to detected linked edges.
  3. Intersect lines to obtain corners.
  - If sufficient care is taken, the points can then be obtained with localization accuracy < 1/10 pixel.



- Rule of thumb
  - Number of constraints should exceed number of unknowns by a factor of five.

  $\Rightarrow$ For 11 parameters of P, at least 28 points should be used.

Camera Calibration: DLT Algorithm

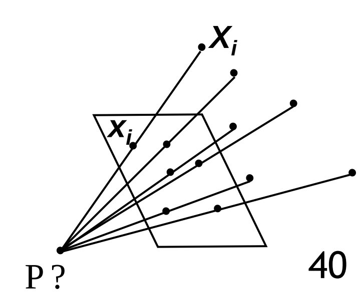
(DLT = "Direct Linear Transform")

$$\lambda \mathbf{x}_i = \mathbf{P}\mathbf{X}_i \qquad \lambda \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} \mathrm{X}_{i,1} \\ \mathrm{X}_{i,2} \\ \mathrm{X}_{i,3} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_1^T \\ \mathbf{P}_2^T \\ \mathbf{P}_3^T \end{bmatrix} \mathbf{X}_i$$

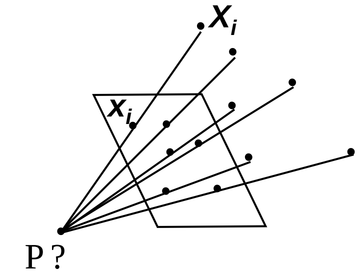
$$\mathbf{x}_i \times \mathbf{P}\mathbf{X}_i = 0$$

$$\begin{bmatrix} 0 & -\mathbf{X}_i^T & y_i \mathbf{X}_i^T \\ \mathbf{X}_i^T & 0 & -x_i \mathbf{X}_i^T \\ -y_i \mathbf{X}_i^T & x_i \mathbf{X}_i^T & 0 \end{bmatrix} \begin{pmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{pmatrix} = 0$$

Only two linearly independent equations



Camera Calibration: DLT Algorithm



$$\begin{bmatrix} 0^T & X_1^T & -y_1 X_1^T \\ X_1^T & 0^T & -x_1 X_1^T \\ \cdots & \cdots & \cdots \\ 0^T & X_n^T & -y_n X_n^T \\ X_n^T & 0^T & -x_n X_n^T \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0$$

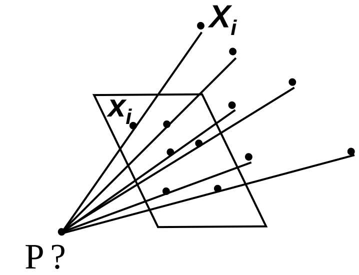
$$Ap = 0$$

Solve using... SVD!

- **Notes**
  - P has 11 degrees of freedom (12 parameters, but scale is arbitrary).
  - One 2D/3D correspondence gives us two linearly independent equations.
  - Homogeneous least squares (similar to homography est.)

  $\Rightarrow$ 5 ½ correspondences needed for a minimal solution.

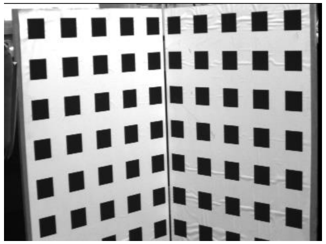
Camera Calibration: DLT Algorithm



$$\begin{bmatrix} 0^T & X_1^T & -y_1 X_1^T \\ X_1^T & 0^T & -x_1 X_1^T \\ \cdots & \cdots & \cdots \\ 0^T & X_n^T & -y_n X_n^T \\ X_n^T & 0^T & -x_n X_n^T \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0 \qquad \mathbf{A}\mathbf{p} = 0$$

- **Notes**
  - For coplanar points that satisfy $\Pi^T X=0$, we will get degenerate solutions $(\Pi,0,0)$, $(0,\Pi,0)$, or $(0,0,\Pi)$.

  $\Rightarrow$ We need calibration points in more than one plane!



Camera Calibration

- Once we've recovered the numerical form of the camera matrix, we still have to figure out the intrinsic and extrinsic parameters
- This is a matrix decomposition problem, not an estimation problem (see F&P sec. 3.2, 3.3)

Camera Calibration: Some Practical Tips

- For numerical reasons, it is important to carry out some data normalization.
  - Translate the image points x_i to the (image) origin and scale them such that their RMS distance to the origin is $\sqrt{2}$.
  - Translate the 3D points X_i to the (world) origin and scale them such that their RMS distance to the origin is $\sqrt{3}$.
  - (This is valid for compact point distributions on calibration objects).
- The DLT algorithm presented here is easy to implement, but there are some more accurate algorithms available (see H&Z sec. 7.2).
- For practical applications, it is also often needed to correct for radial distortion. Algorithms for this can be found in H&Z sec. 7.4, or F&P sec. 3.3.

Topics of This Lecture

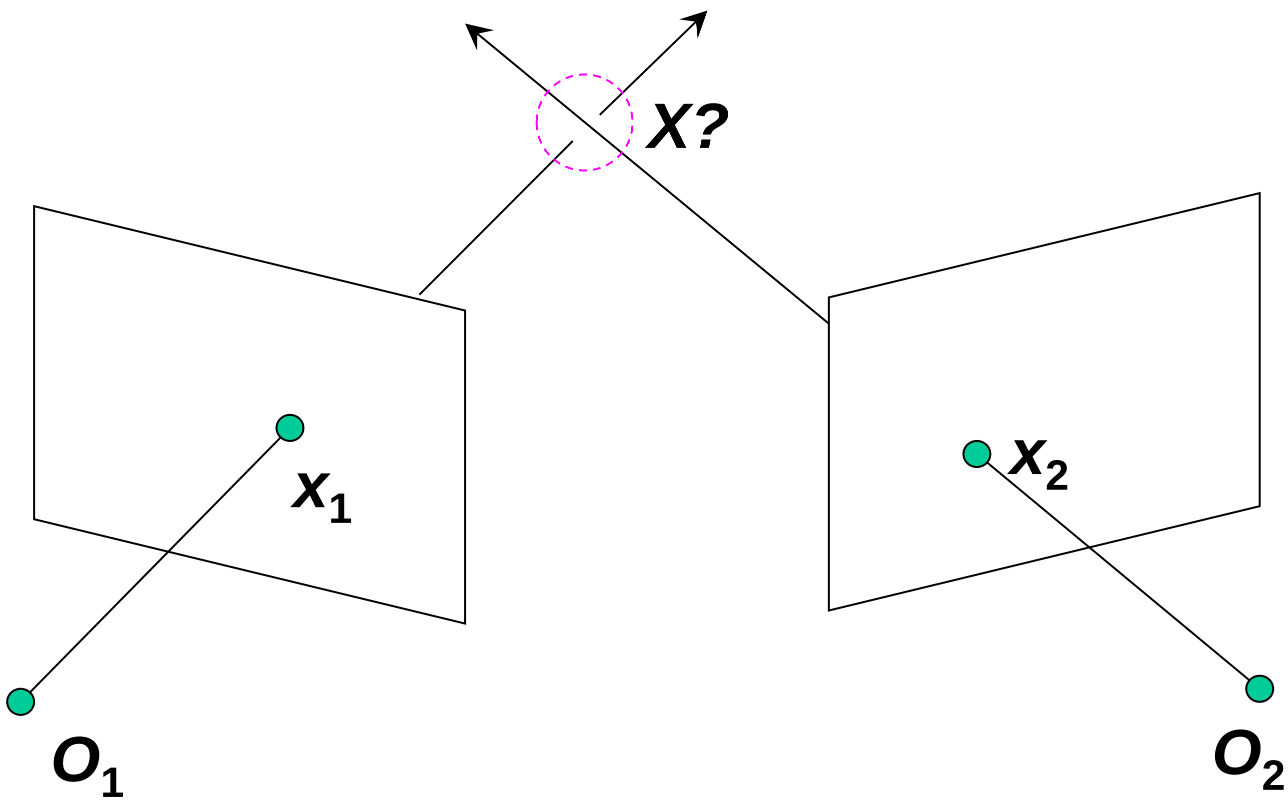
- Camera Calibration
  - Camera parameters
  - Calibration procedure
- Revisiting Epipolar Geometry
  - Triangulation
  - Calibrated case: Essential matrix
  - Uncalibrated case: Fundamental matrix
  - Weak calibration
  - Epipolar Transfer
- Active Stereo
  - Laser scanning
  - Kinect sensor

Two-View Geometry

- Scene geometry (structure):
  - Given corresponding points in two or more images, where is the pre-image of these points in 3D?
- Correspondence (stereo matching):
  - Given a point in just one image, how does it constrain the position of the corresponding point x' in another image?
- Camera geometry (motion):
  - Given a set of corresponding points in two images, what are the cameras for the two views?

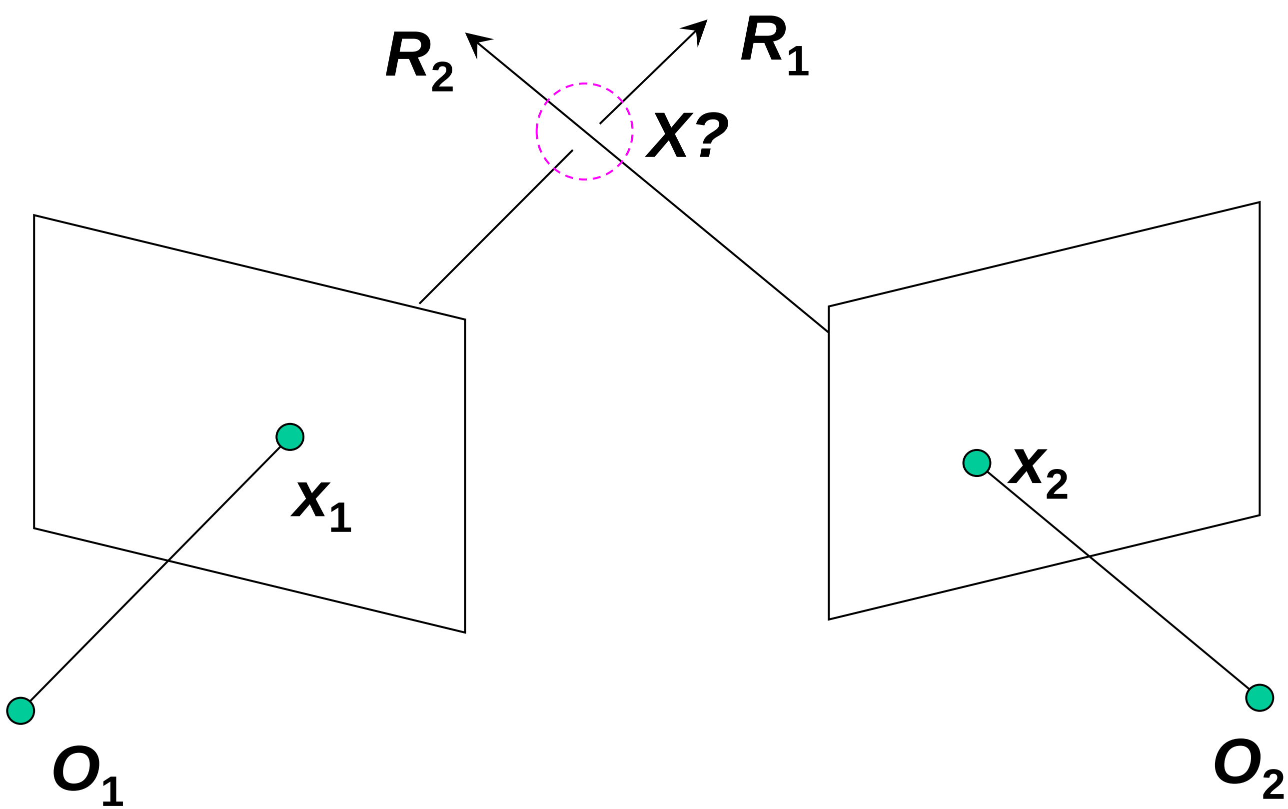
Revisiting Triangulation

- Given projections of a 3D point in two or more images (with known camera matrices), find the coordinates of the point



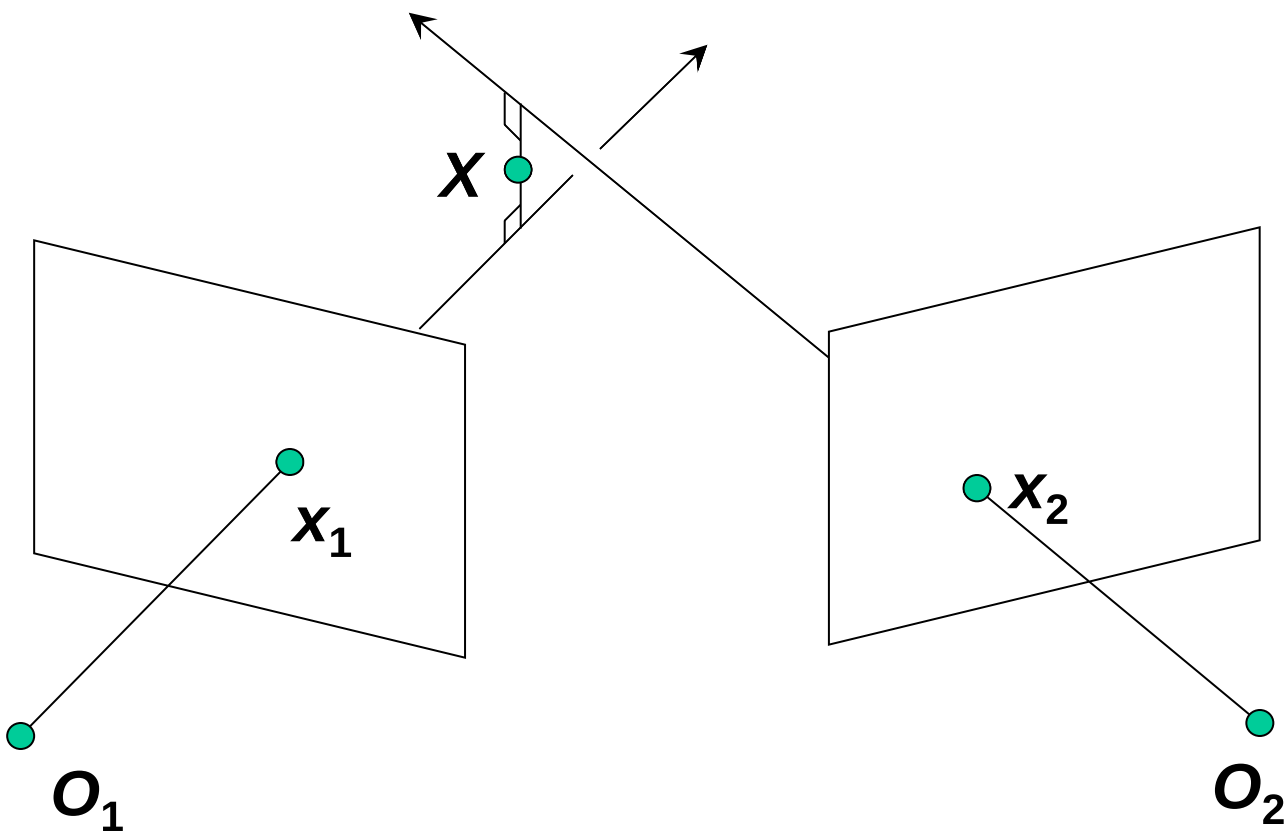
Revisiting Triangulation

- We want to intersect the two visual rays corresponding to x_1 and x_2, but because of noise and numerical errors, they will never meet exactly. How can this be done?



Triangulation: 1) Geometric Approach

- Find shortest segment connecting the two viewing rays and let X be the midpoint of that segment.



Triangulation: 2)Linear Algebraic Approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}] \mathrm{P}_1 \mathrm{X} = 0$$

$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}] \mathrm{P}_2 \mathrm{X} = 0$$

Cross product as matrix multiplication:

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times] \mathbf{b}$$

Triangulation: 2) Linear Algebraic Approach

$$\lambda_1 \mathrm{x}_1 = \mathrm{P}_1 \mathrm{X} \qquad \mathrm{x}_1 \times \mathrm{P}_1 \mathrm{X} = 0 \qquad [\mathrm{x}_{1\times}] \mathrm{P}_1 \mathrm{X} = 0$$

$$\lambda_2 \mathrm{x}_2 = \mathrm{P}_2 \mathrm{X} \qquad \mathrm{x}_2 \times \mathrm{P}_2 \mathrm{X} = 0 \qquad [\mathrm{x}_{2\times}] \mathrm{P}_2 \mathrm{X} = 0$$

Two independent equations each in terms of three unknown entries of X

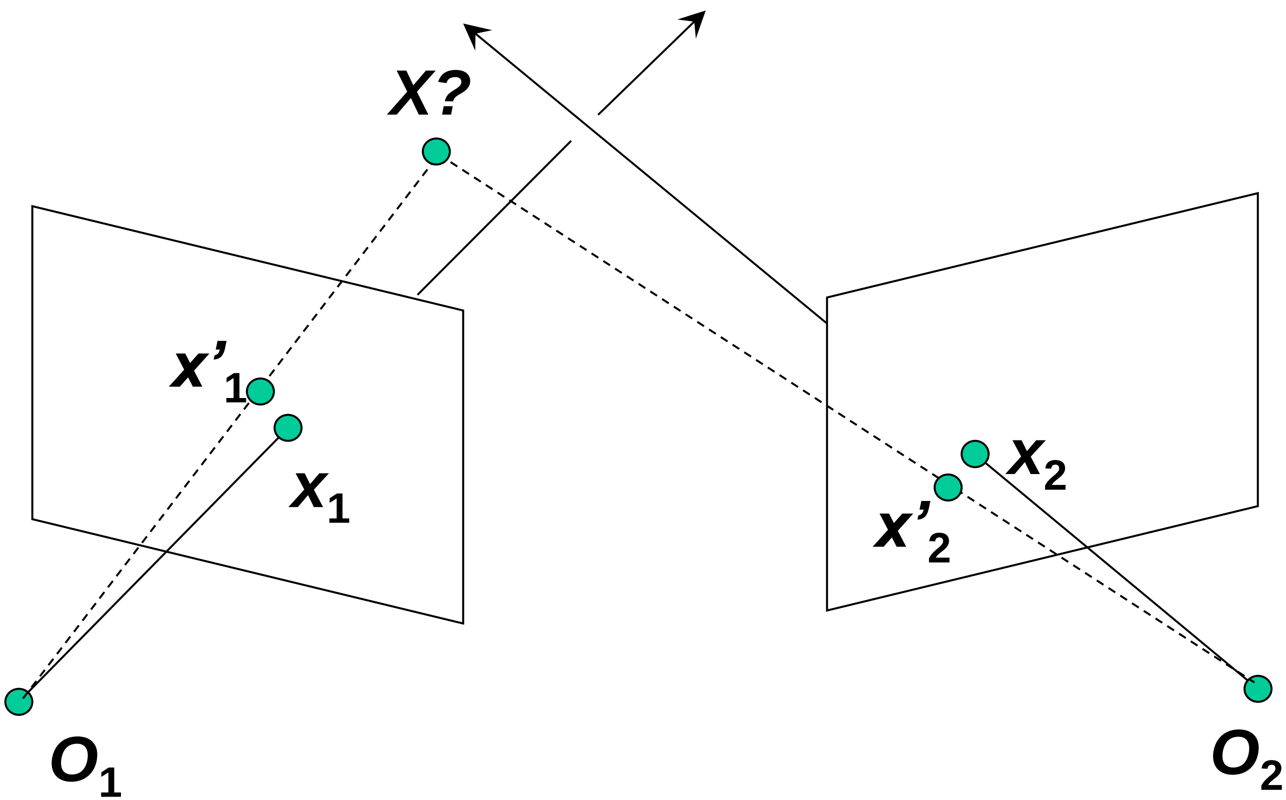
$\Rightarrow$ Stack them and solve using SVD!

- **This approach is often preferable to the geometric approach, since it nicely generalizes to multiple cameras.**

Triangulation: 3) Nonlinear Approach

- **Find X that minimizes**

$$d^2(x_1, P_1X) + d^2(x_2, P_2X)$$



Triangulation: 3) Nonlinear Approach

- Find X that minimizes

$$d^2(x_1, P_1 X) + d^2(x_2, P_2 X)$$

- This approach is the most accurate, but unlike the other two methods, it doesn't have a closed-form solution.

- Iterative algorithm
  - Initialize with linear estimate.
  - Optimize with Gauss-Newton or Levenberg-Marquardt (see F&P sec. 3.1.2 or H&Z Appendix 6).

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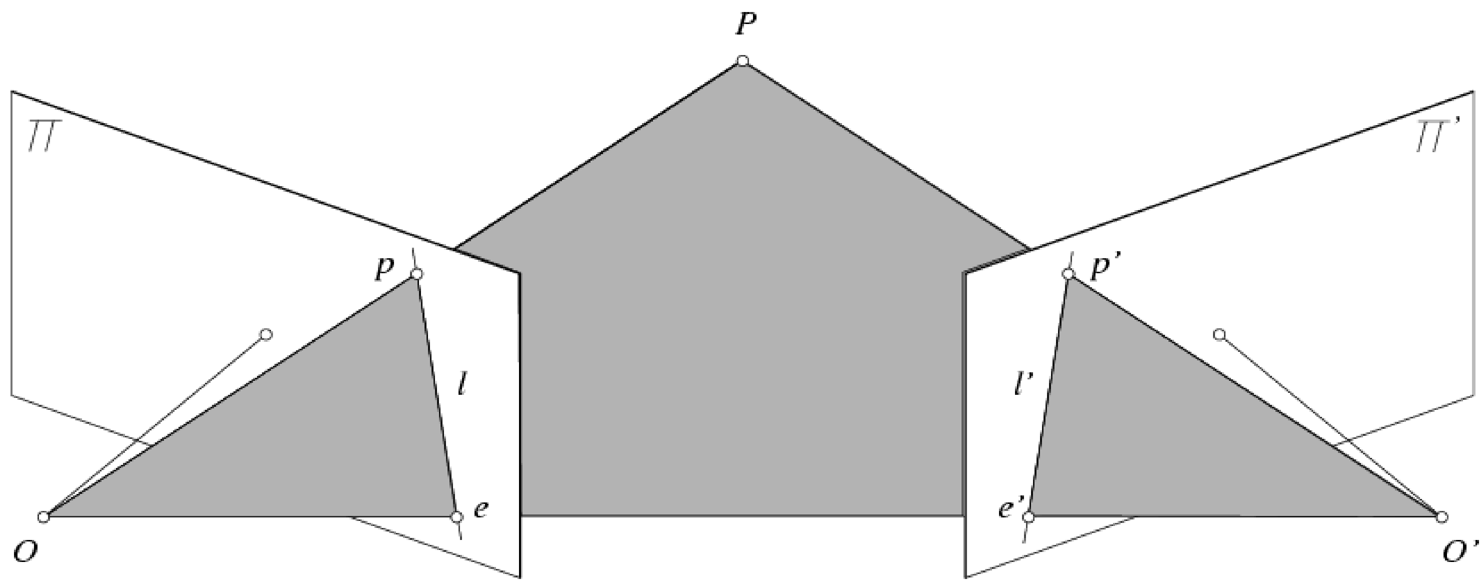
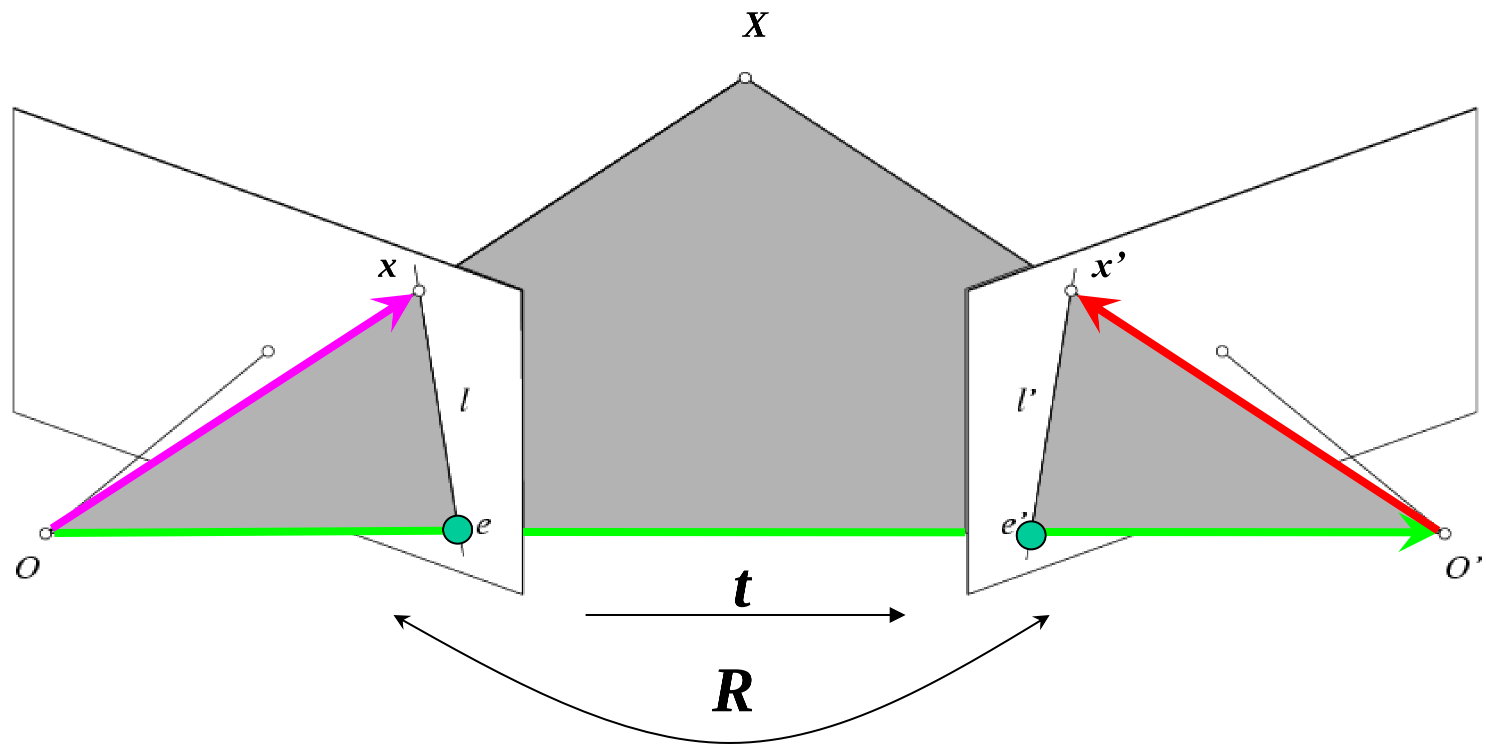
Revisiting Epipolar Geometry



- **Let's look again at the epipolar constraint**
  - For the calibrated case (but in homogenous coordinates)
  - For the uncalibrated case

Epipolar Geometry: Calibrated Case

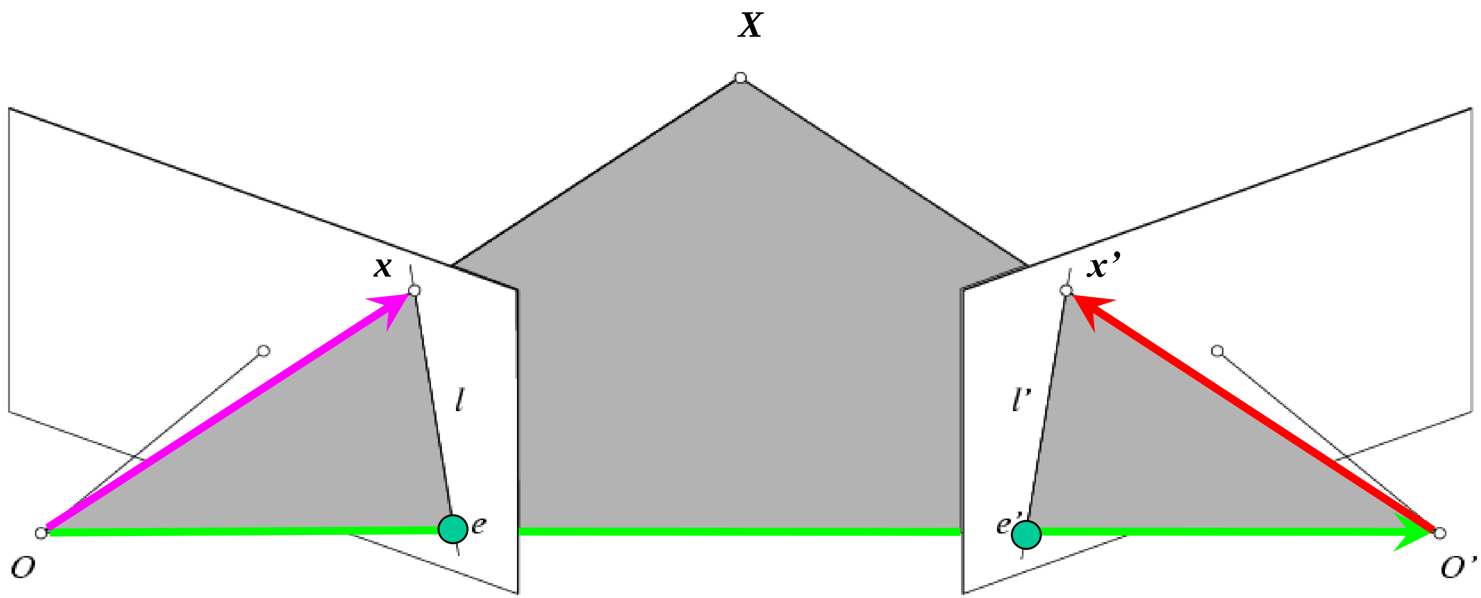


Camera matrix: $[\mathbf{I}|0]$
$X = (u, v, w, 1)^T$
$x = (u, v, w)^T$

Camera matrix: $[R^T \mid -R^T t]$
Vector x' in second coord. system has coordinates Rx' in the first one.

The vectors x, t, and Rx' are coplanar

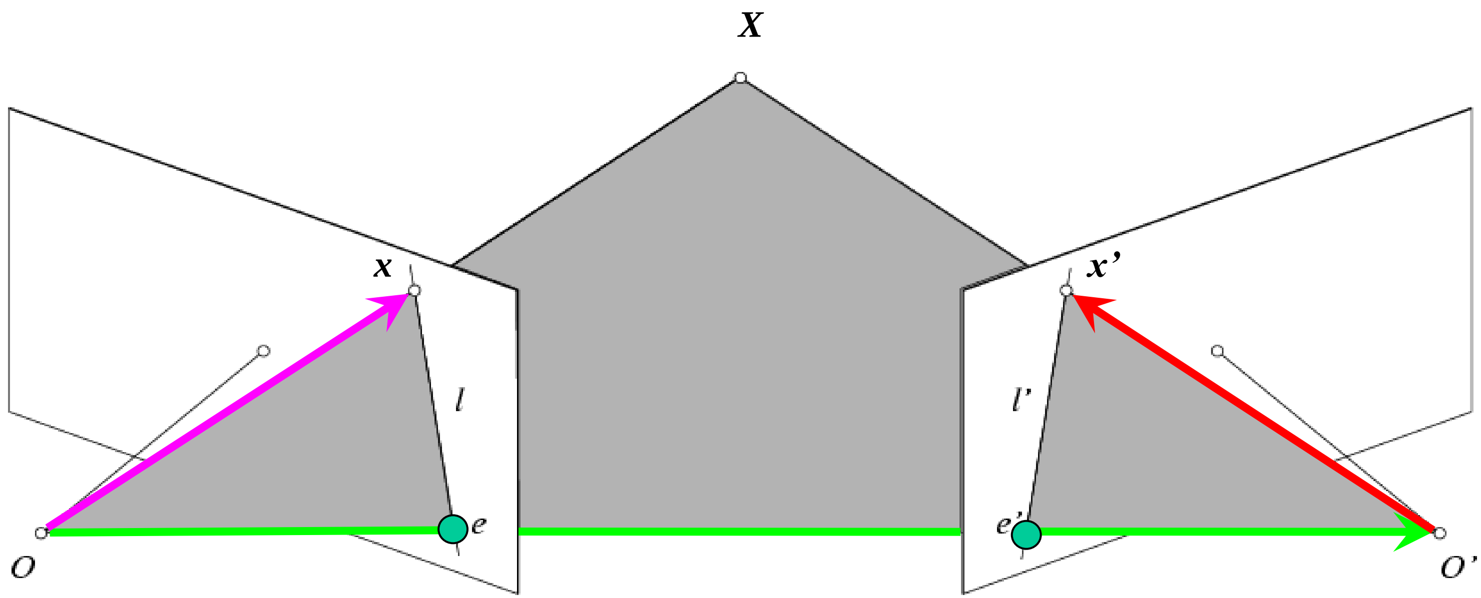
Epipolar Geometry: Calibrated Case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_\times]R$$

Essential Matrix (Longuet-Higgins, 1981)

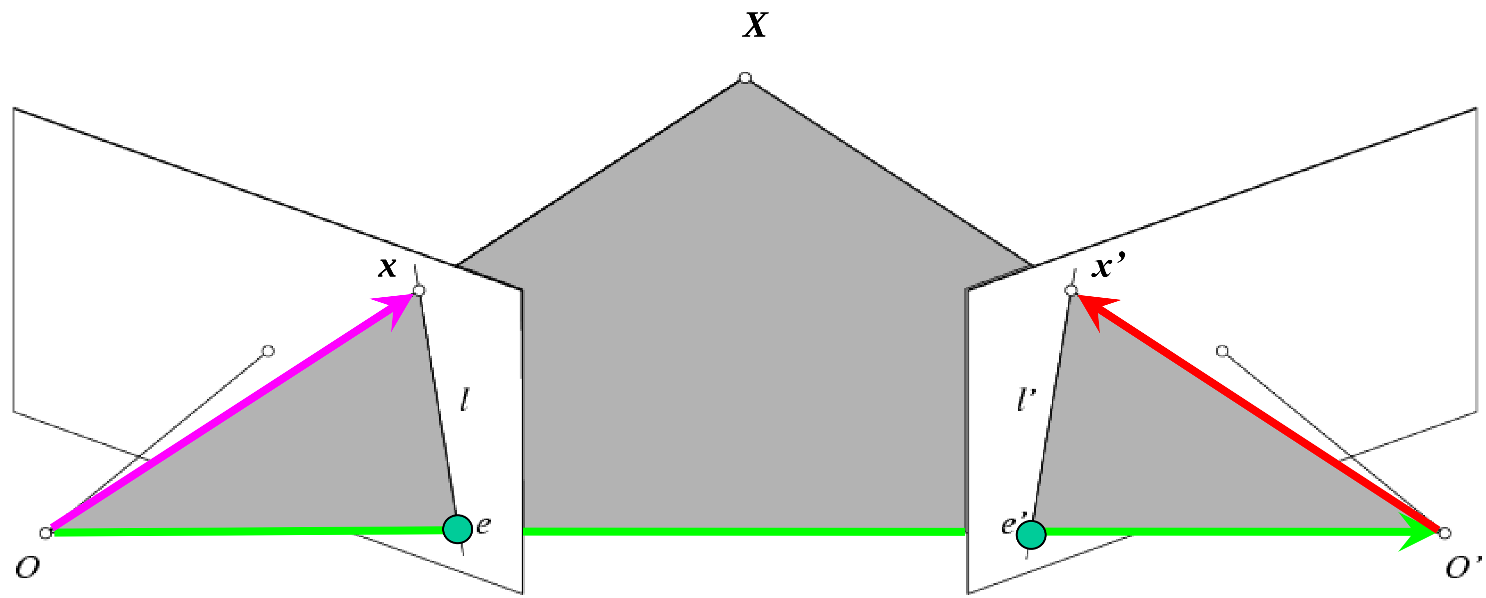
Epipolar Geometry: Calibrated Case



$$x \cdot [t \times (Rx')] = 0 \quad \Rightarrow \quad x^T E x' = 0 \quad \text{with} \quad E = [t_\times]R$$

- ***E x'*** **is the epipolar line associated with *x'* (*l* = *E x'*)**
- ***E^T^x*** **is the epipolar line associated with *x* (*l'* = *E^T^x*)**
- ***E e'* = 0 and *E^T^e* = *0***
- ***E*** **is singular (rank two)**
- ***E*** **has five degrees of freedom (up to scale)**

Epipolar Geometry: Uncalibrated Case



- **The calibration matrices *K* and *K'* of the two cameras are unknown**
- **We can write the epipolar constraint in terms of *unknown* normalized coordinates:**

$$\hat{x}^T E \hat{x}' = 0 \qquad\qquad x = K\hat{x}, \quad x' = K'\hat{x}'$$

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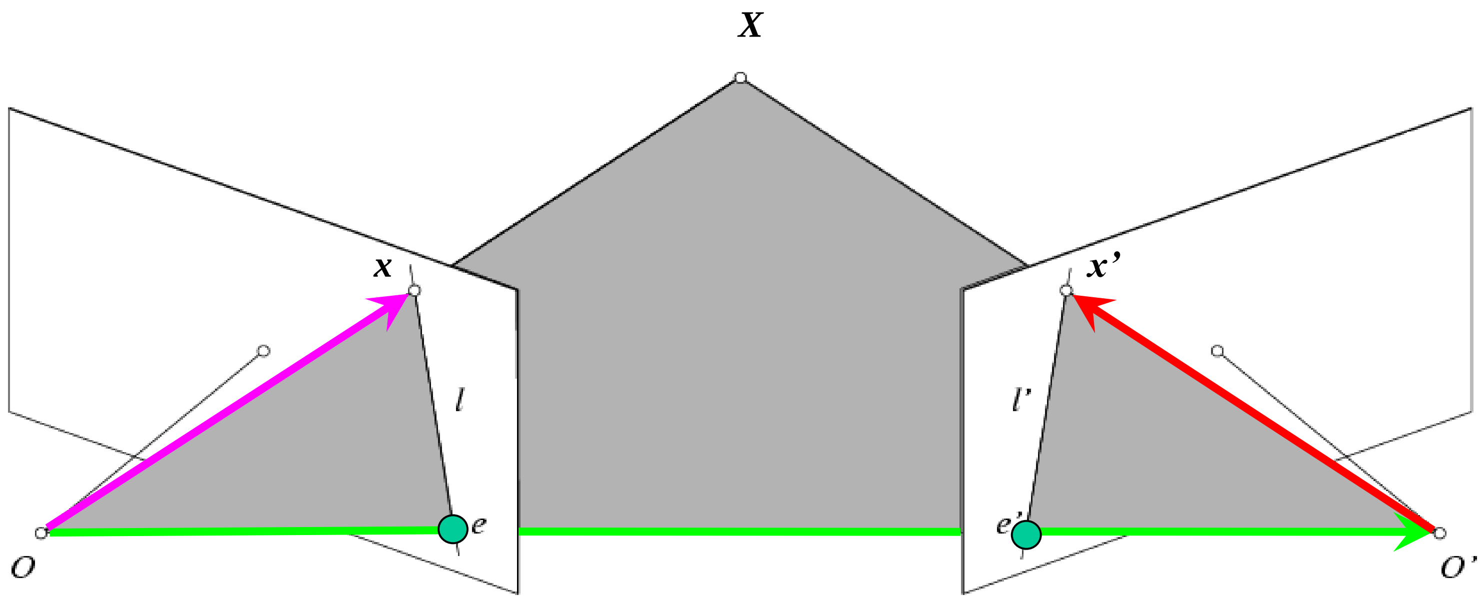
Epipolar Geometry: Uncalibrated Case



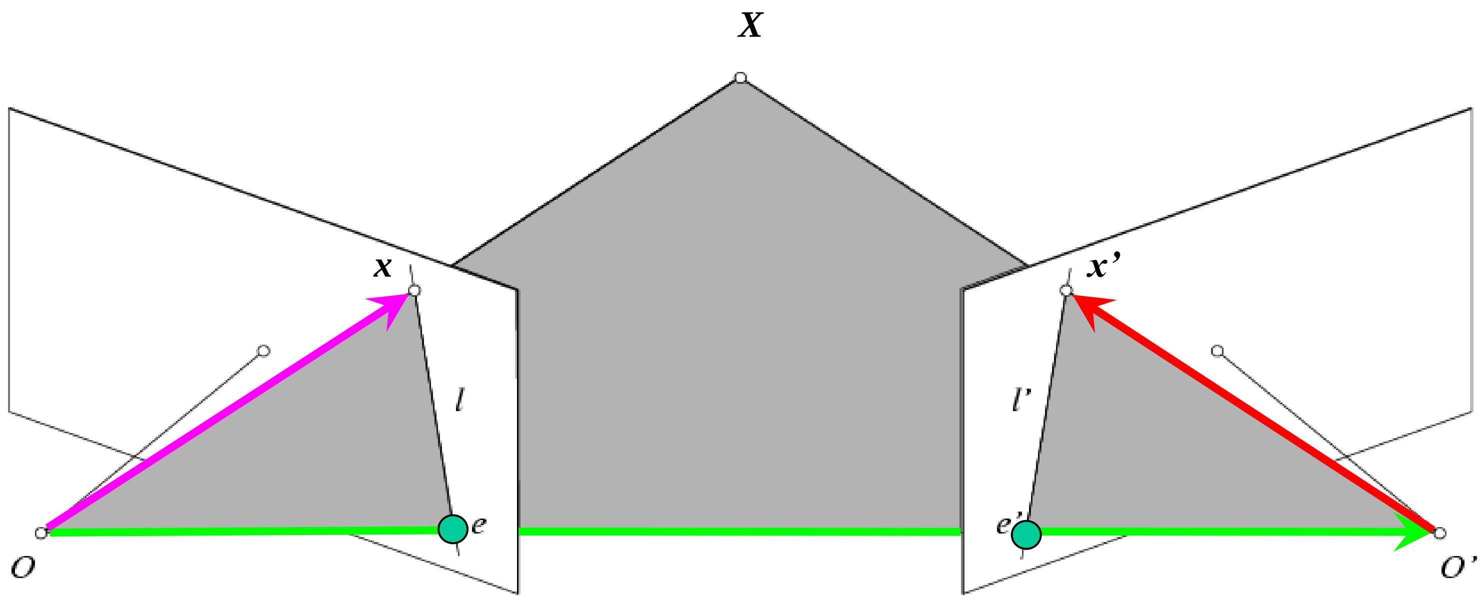
$$\hat{x}^T E \hat{x}' = 0 \quad \Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

$$x = K \hat{x}$$

$$x' = K' \hat{x}'$$

Fundamental Matrix
(Faugeras and Luong, 1992)

Epipolar Geometry: Uncalibrated Case



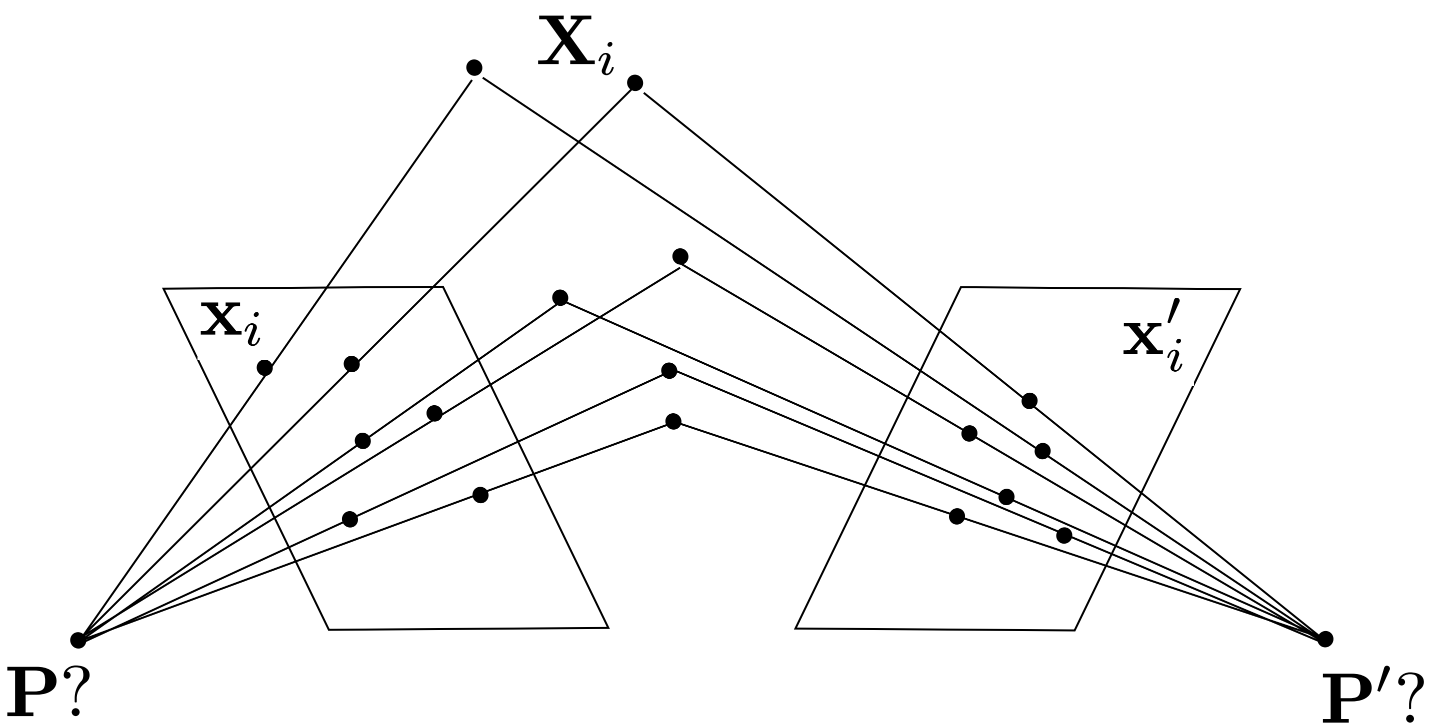
$$\hat{x}^T E \hat{x}' = 0 \quad \Rightarrow \quad x^T F x' = 0 \quad \text{with} \quad F = K^{-T} E K'^{-1}$$

- *F x'* is the epipolar line associated with *x'* (*l* = *F x'*)
- $F^T x$ is the epipolar line associated with *x* (*l'* = $F^T x$)
- *F e'* = 0 and $F^T e = 0$
- *F* is singular (rank two)
- *F* has seven degrees of freedom

Slide credit: Svetlana Lazebnik

Estimating the Fundamental Matrix

- The Fundamental matrix defines the epipolar geometry between two uncalibrated cameras.
- How can we estimate F from an image pair?
  - We need correspondences…



The Eight-Point Algorithm

$\boldsymbol{x} = (\boldsymbol{u}, \boldsymbol{v}, \mathbf{1})^T, \quad \boldsymbol{x}' = (\boldsymbol{u}', \boldsymbol{v}', \mathbf{1})^T$

$$(u, v, 1)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix}\begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \quad \Rightarrow \quad [u'u, u'v, u', uv', vv', v', u, v, 1]\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

- **Taking 8 correspondences:**

$$\begin{bmatrix} u'_1u_1 & u'_1v_1 & u'_1 & u_1v'_1 & v_1v'_1 & v'_1 & u_1 & v_1 & 1 \\ u'_2u_2 & u'_2v_2 & u'_2 & u_2v'_2 & v_2v'_2 & v'_2 & u_2 & v_2 & 1 \\ u'_3u_3 & u'_3v_3 & u'_3 & u_3v'_3 & v_3v'_3 & v'_3 & u_3 & v_3 & 1 \\ u'_4u_4 & u'_4v_4 & u'_4 & u_4v'_4 & v_4v'_4 & v'_4 & u_4 & v_4 & 1 \\ u'_5u_5 & u'_5v_5 & u'_5 & u_5v'_5 & v_5v'_5 & v'_5 & u_5 & v_5 & 1 \\ u'_6u_6 & u'_6v_6 & u'_6 & u_6v'_6 & v_6v'_6 & v'_6 & u_6 & v_6 & 1 \\ u'_7u_7 & u'_7v_7 & u'_7 & u_7v'_7 & v_7v'_7 & v'_7 & u_7 & v_7 & 1 \\ u'_8u_8 & u'_8v_8 & u'_8 & u_8v'_8 & v_8v'_8 & v'_8 & u_8 & v_8 & 1 \end{bmatrix}\begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{Af} = \mathbf{0}$$

This minimizes:

$$\sum_{i=1}^{N} (x_i^T F x_i')^2$$

Solve using… SVD!

Excursion: Properties of SVD

- **Frobenius norm**
  - Generalization of the Euclidean norm to matrices

$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2} = \sqrt{\sum_{i=1}^{\min(m,n)} \sigma_i^2}$$

- **Partial reconstruction property of SVD**
  - Let $\sigma_i \; i=1,\ldots,N$ be the singular values of A.
  - Let $A_p = U_p D_p V_p^T$ be the reconstruction of A when we set $\sigma_{p+1}, \ldots, \sigma_N$ to zero.
  - Then $A_p = U_p D_p V_p^T$ is the best rank-p approximation of A in the sense of the Frobenius norm (i.e. the best least-squares approximation).

The Eight-Point Algorithm

- **Problem with noisy data**
  - The solution will usually not fulfill the constraint that F only has rank 2.

  $\Rightarrow$ *There will be no epipoles through which all epipolar lines pass!*

- **Enforce the rank-2 constraint using SVD**

$$F \overset{\text{SVD}}{=} \boldsymbol{UDV}^T = \boldsymbol{U} \begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix} \begin{bmatrix} v_{11} & \cdots & v_{13} \\ \vdots & \ddots & \vdots \\ v_{31} & \cdots & v_{33} \end{bmatrix}^T$$

Set d_{33} to zero and reconstruct F

- **As we have just seen, this provides the best least-squares approximation to the rank-2 solution.**

Problem with the Eight-Point Algorithm

- **In practice, this often looks as follows:**

$$\begin{bmatrix} u'_1u_1 & u'_1v_1 & u'_1 & u_1v'_1 & v_1v'_1 & v'_1 & u_1 & v_1 & 1 \\ u'_2u_2 & u'_2v_2 & u'_2 & u_2v'_2 & v_2v'_2 & v'_2 & u_2 & v_2 & 1 \\ u'_3u_3 & u'_3v_3 & u'_3 & u_3v'_3 & v_3v'_3 & v'_3 & u_3 & v_3 & 1 \\ u'_4u_4 & u'_4v_4 & u'_4 & u_4v'_4 & v_4v'_4 & v'_4 & u_4 & v_4 & 1 \\ u'_5u_5 & u'_5v_5 & u'_5 & u_5v'_5 & v_5v'_5 & v'_5 & u_5 & v_5 & 1 \\ u'_6u_6 & u'_6v_6 & u'_6 & u_6v'_6 & v_6v'_6 & v'_6 & u_6 & v_6 & 1 \\ u'_7u_7 & u'_7v_7 & u'_7 & u_7v'_7 & v_7v'_7 & v'_7 & u_7 & v_7 & 1 \\ u'_8u_8 & u'_8v_8 & u'_8 & u_8v'_8 & v_8v'_8 & v'_8 & u_8 & v_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Problem with the Eight-Point Algorithm

- **In practice, this often looks as follows:**

$$\begin{bmatrix} 250906.36 & 183269.57 & 921.81 & 200931.10 & 146766.13 & 738.21 & 272.19 & 198.81 & 1 \\ 2692.28 & 131633.03 & 176.27 & 6196.73 & 302975.59 & 405.71 & 15.27 & 746.79 & 1 \\ 416374.23 & 871684.30 & 935.47 & 408110.89 & 854384.92 & 916.90 & 445.10 & 931.81 & 1 \\ 191183.60 & 171759.40 & 410.27 & 416435.62 & 374125.90 & 893.65 & 465.99 & 418.65 & 1 \\ 48988.86 & 30401.76 & 57.89 & 298604.57 & 185309.58 & 352.87 & 846.22 & 525.15 & 1 \\ 164786.04 & 546559.67 & 813.17 & 1998.37 & 6628.15 & 9.86 & 202.65 & 672.14 & 1 \\ 116407.01 & 2727.75 & 138.89 & 169941.27 & 3982.21 & 202.77 & 838.12 & 19.64 & 1 \\ 135384.58 & 75411.13 & 198.72 & 411350.03 & 229127.78 & 603.79 & 681.28 & 379.48 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ **Poor numerical conditioning**

$\Rightarrow$ **Can be fixed by rescaling the data**

The Normalized Eight-Point Algorithm

1. Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels.
2. Use the eight-point algorithm to compute *F* from the normalized points.
3. Enforce the rank-2 constraint using SVD.

$$F \overset{\text{SVD}}{=} UDV^T = U \begin{bmatrix} d_{11} & & \\ & d_{22} & \\ & & d_{33} \end{bmatrix} \begin{bmatrix} v_{11} & \cdots & v_{13} \\ \vdots & \ddots & \vdots \\ v_{31} & \cdots & v_{33} \end{bmatrix}^T$$

Set d_{33} to zero and reconstruct F

4. Transform fundamental matrix back to original units: if T and T' are the normalizing transformations in the two images, than the fundamental matrix in original coordinates is $T^T F T'$.

The Eight-Point Algorithm

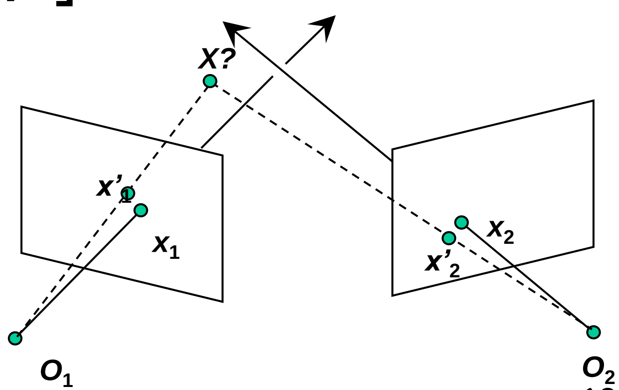
- **Meaning of error** $\sum_{i=1}^{N}(x_i^T F x_i')^2$ **:**

  Sum of Euclidean distances between points x_i and epipolar lines $F x'_i$ (or points x'_i and epipolar lines $F^T x_i$), multiplied by a scale factor

- **Nonlinear approach: minimize**

$$\sum_{i=1}^{N}\left[\mathrm{d}^2(x_i, F x_i') + \mathrm{d}^2(x_i', F^T x_i)\right]$$

  - **Similar to nonlinear minimization approach for triangulation.**
  - **Iterative approach (Gauss-Newton, Levenberg-Marquardt,…)**



Comparison of Estimation Algorithms

	8-point	Normalized 8-point	Nonlinear least squares
Av. Dist. 1	2.33 pixels	0.92 pixel	0.86 pixel
Av. Dist. 2	2.18 pixels	0.85 pixel	0.80 pixel

Slide credit: Svetlana Lazebnik

3D Reconstruction with Weak Calibration

- Want to estimate world geometry without requiring calibrated cameras.
- Many applications:
  - Archival videos
  - Photos from multiple unrelated users
  - Dynamic camera system
- Main idea:
  - Estimate epipolar geometry from a (redundant) set of point correspondences between two uncalibrated cameras.

Slide credit: Kristen Grauman

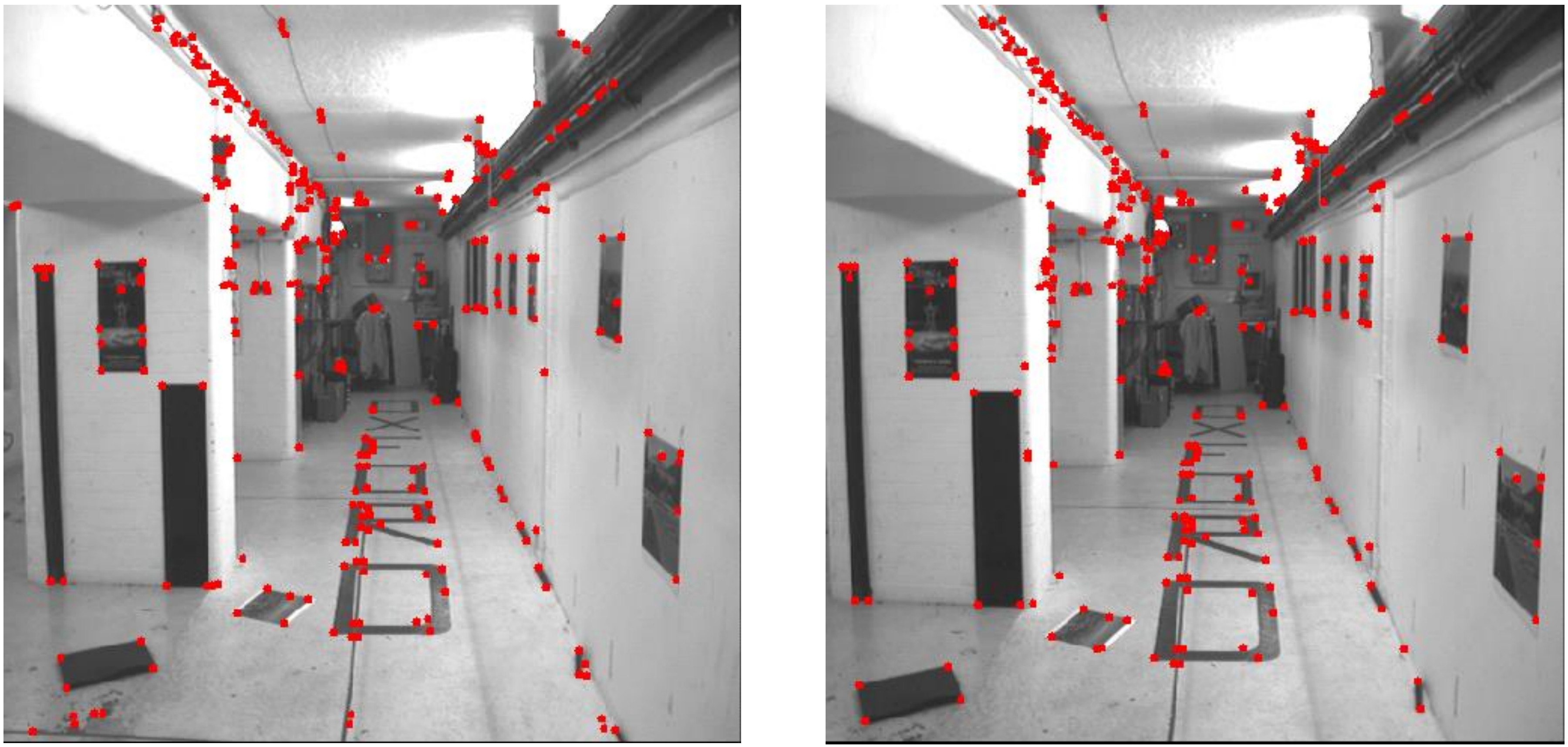
Stereo Pipeline with Weak Calibration

- **So, where to start with uncalibrated cameras?**
  - **Need to find fundamental matrix F *and* the correspondences (pairs of points (u',v') $\leftrightarrow$ (u,v)).**

- **Procedure**
  1. **Find interest points in both images**
  2. **Compute correspondences**
  3. **Compute epipolar geometry**
  4. **Refine**

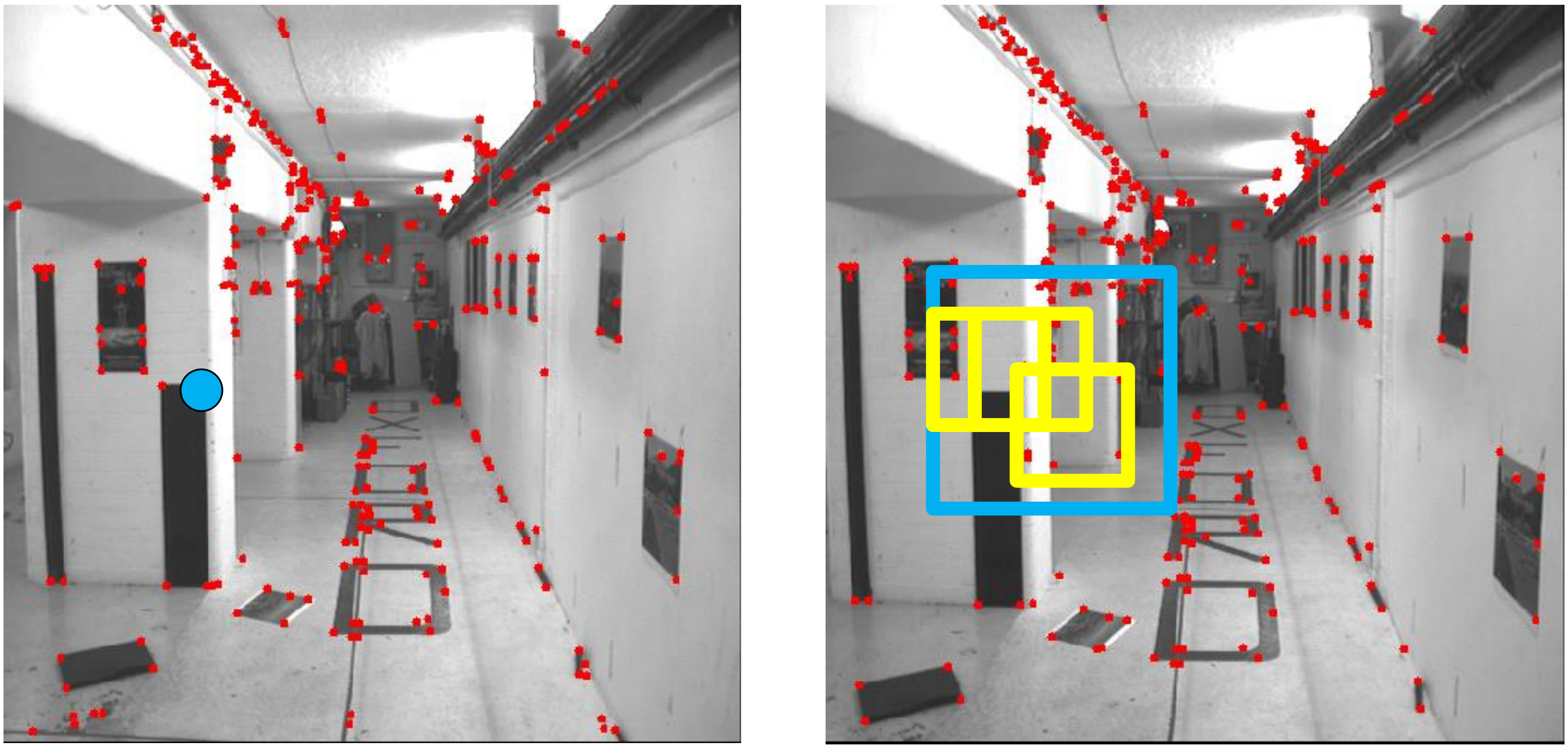
Stereo Pipeline with Weak Calibration

1. Find interest points (e.g. Harris corners)

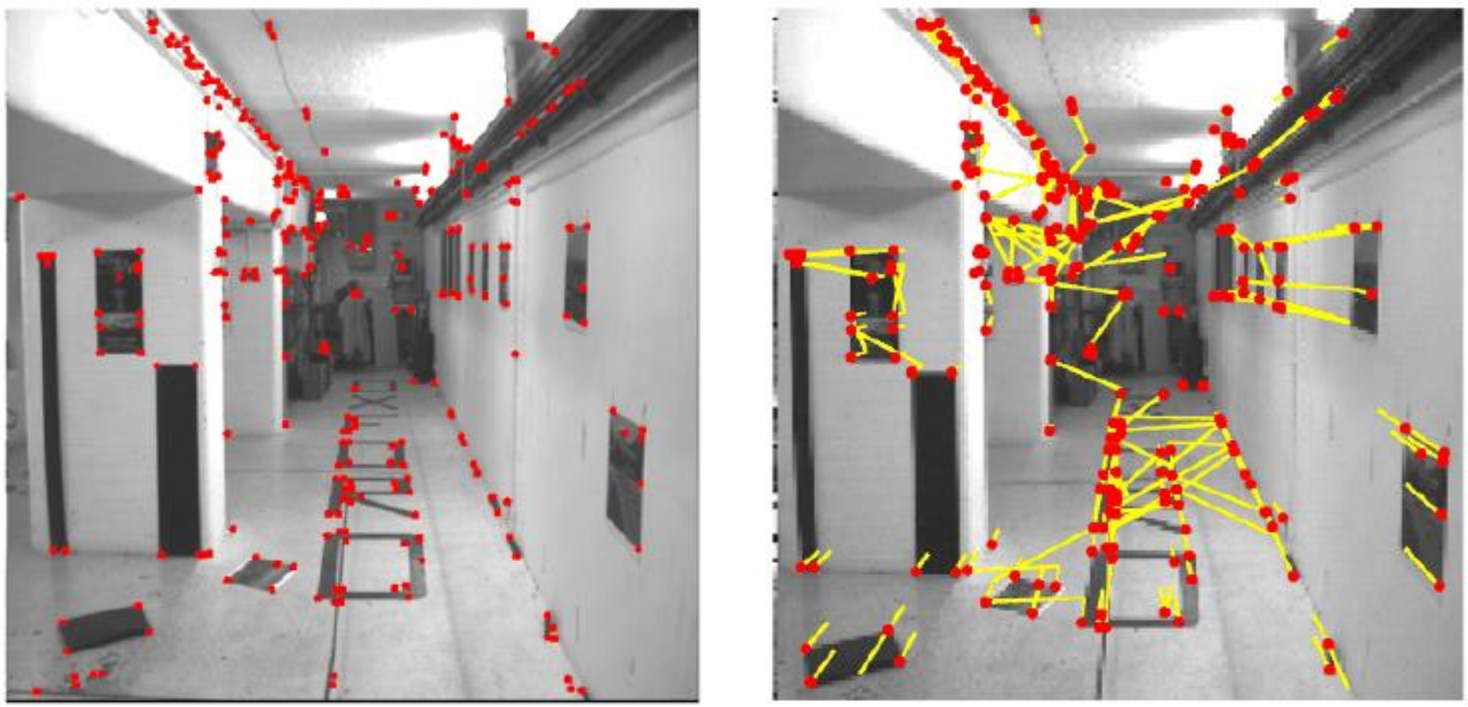


Stereo Pipeline with Weak Calibration

2. Match points using only proximity



Putative Matches based on Correlation Search



- Many wrong matches (10-50%), but enough to compute F

Example from Andrew Zisserman

RANSAC for Robust Estimation of F

- Select random sample of correspondences
- Compute F using them
  - This determines epipolar constraint
- Evaluate amount of support - number of inliers within threshold distance of epipolar line
- Choose F with most support (#inliers)

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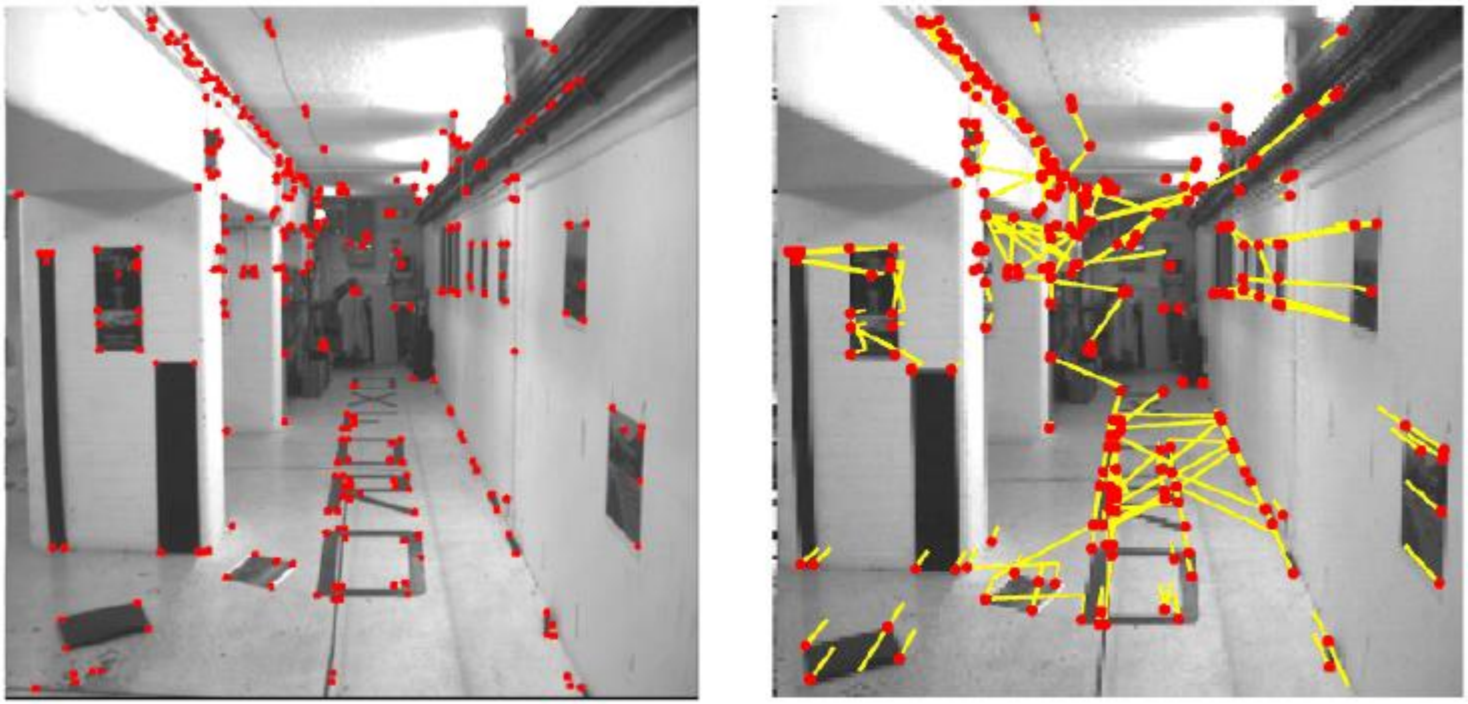
Putative Matches based on Correlation Search

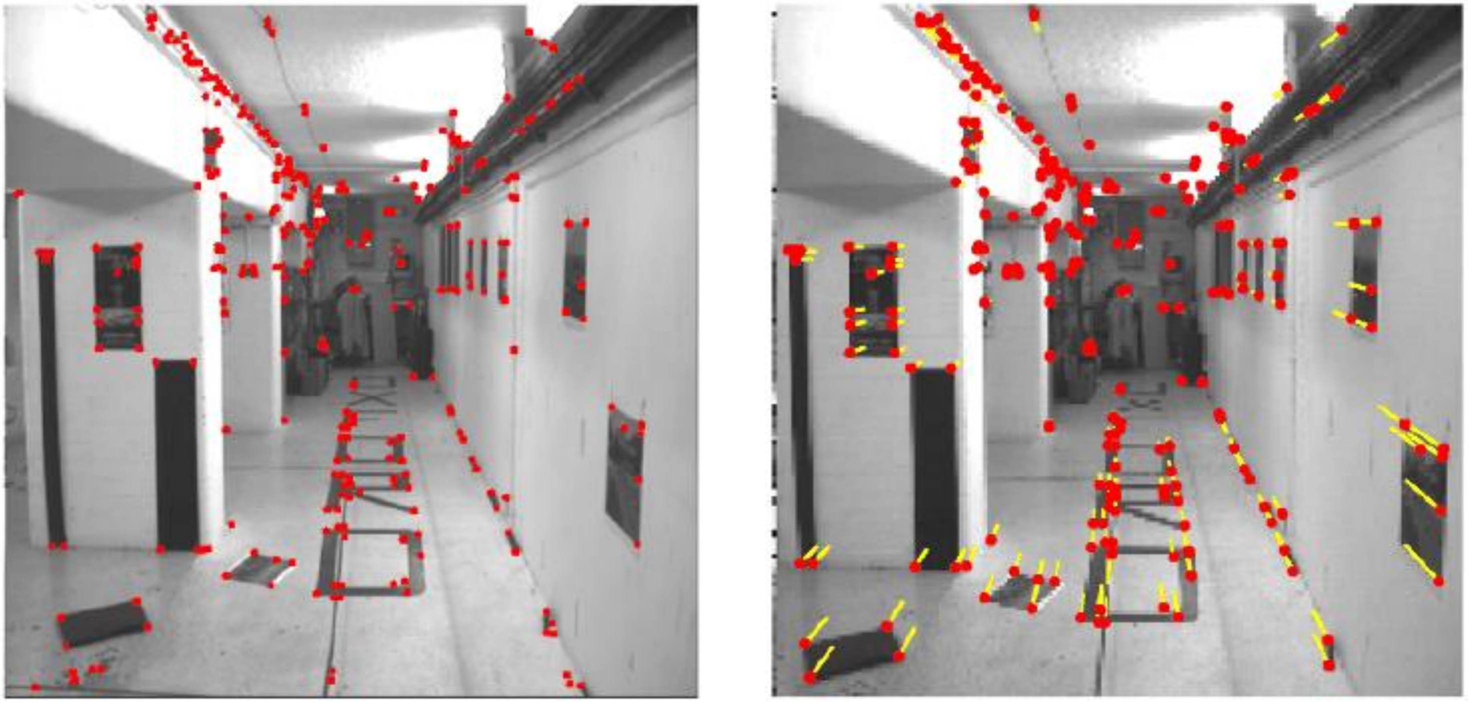


- Many wrong matches (10-50%), but enough to compute F

Example from Andrew Zisserman

Pruned Matches

- Correspondences consistent with epipolar geometry



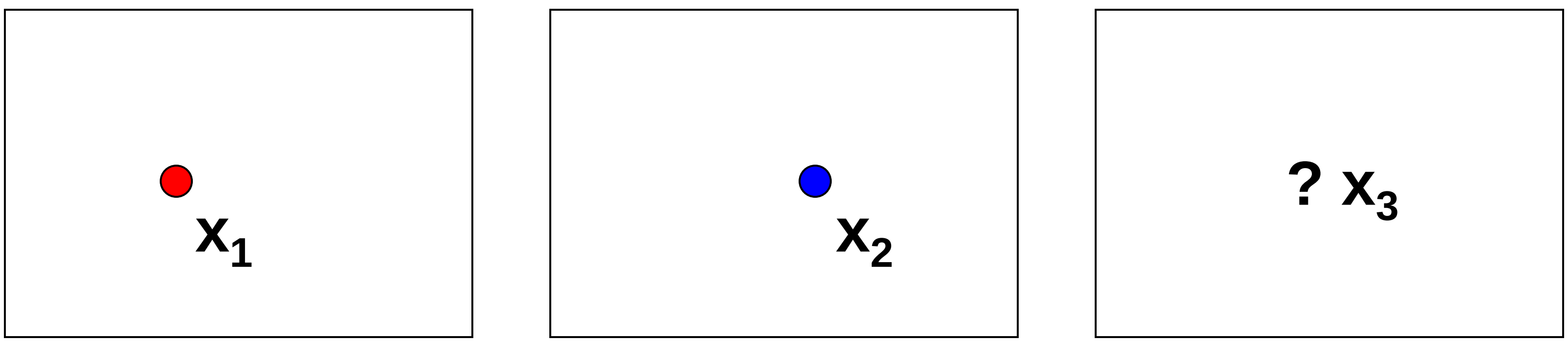
Example from Andrew Zisserman

Resulting Epipolar Geometry

Example from Andrew Zisserman

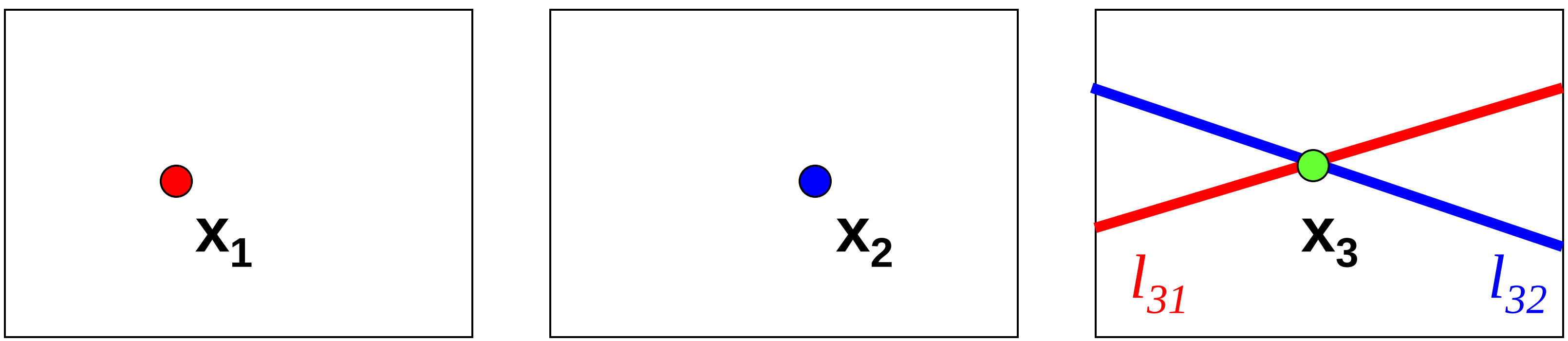
Epipolar Transfer

- Assume the epipolar geometry is known
- Given projections of the same point in two images, how can we compute the projection of that point in a third image?



Extension: Epipolar Transfer

- Assume the epipolar geometry is known
- Given projections of the same point in two images, how can we compute the projection of that point in a third image?



$$l_{31} = F^T{}_{13}\, x_1$$

$$l_{32} = F^T{}_{23}\, x_2$$

When does epipolar transfer fail?

Slide credit: Svetlana Lazebnik

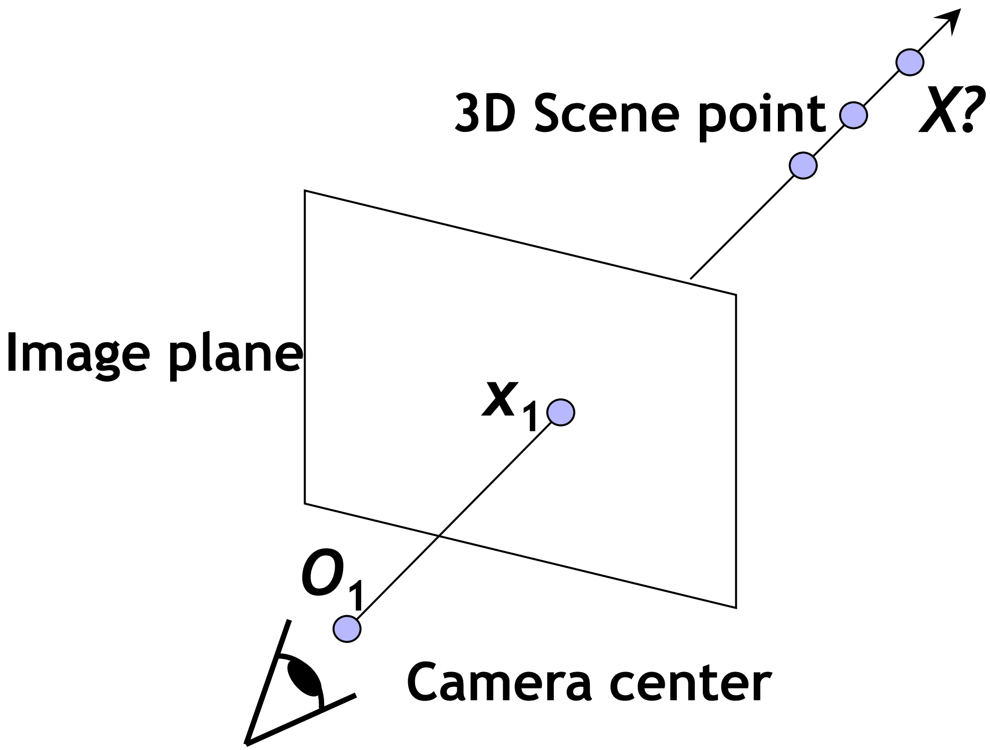
Topics of This Lecture

- Camera Calibration
  - Camera parameters
  - Calibration procedure
- Revisiting Epipolar Geometry
  - Triangulation
  - Calibrated case: Essential matrix
  - Uncalibrated case: Fundamental matrix
  - Weak calibration
  - Epipolar Transfer
- Active Stereo
  - Laser scanning
  - Kinect sensor

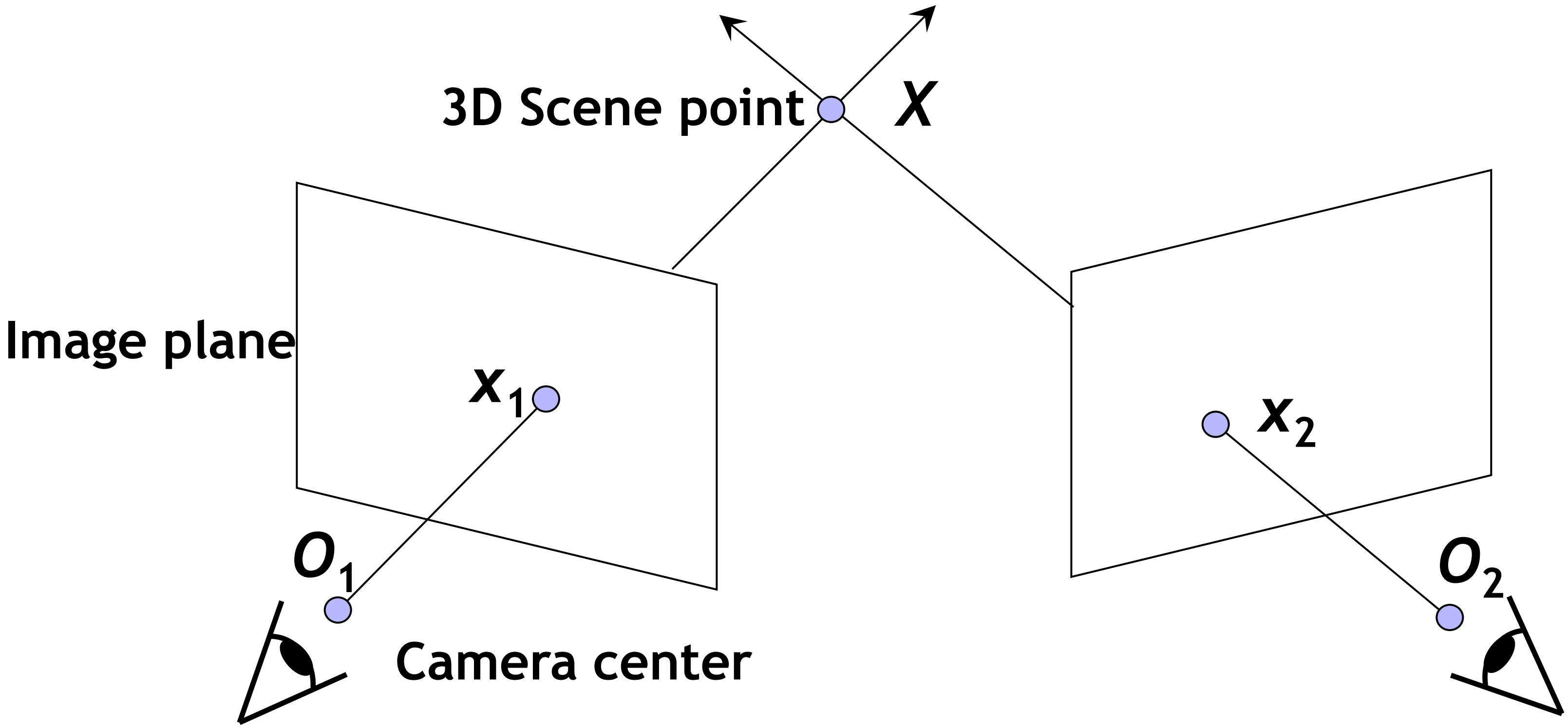
Microsoft Kinect – How Does It Work?

- Built-in IR projector
- IR camera for depth
- Regular camera for color

Recall: Optical Triangulation

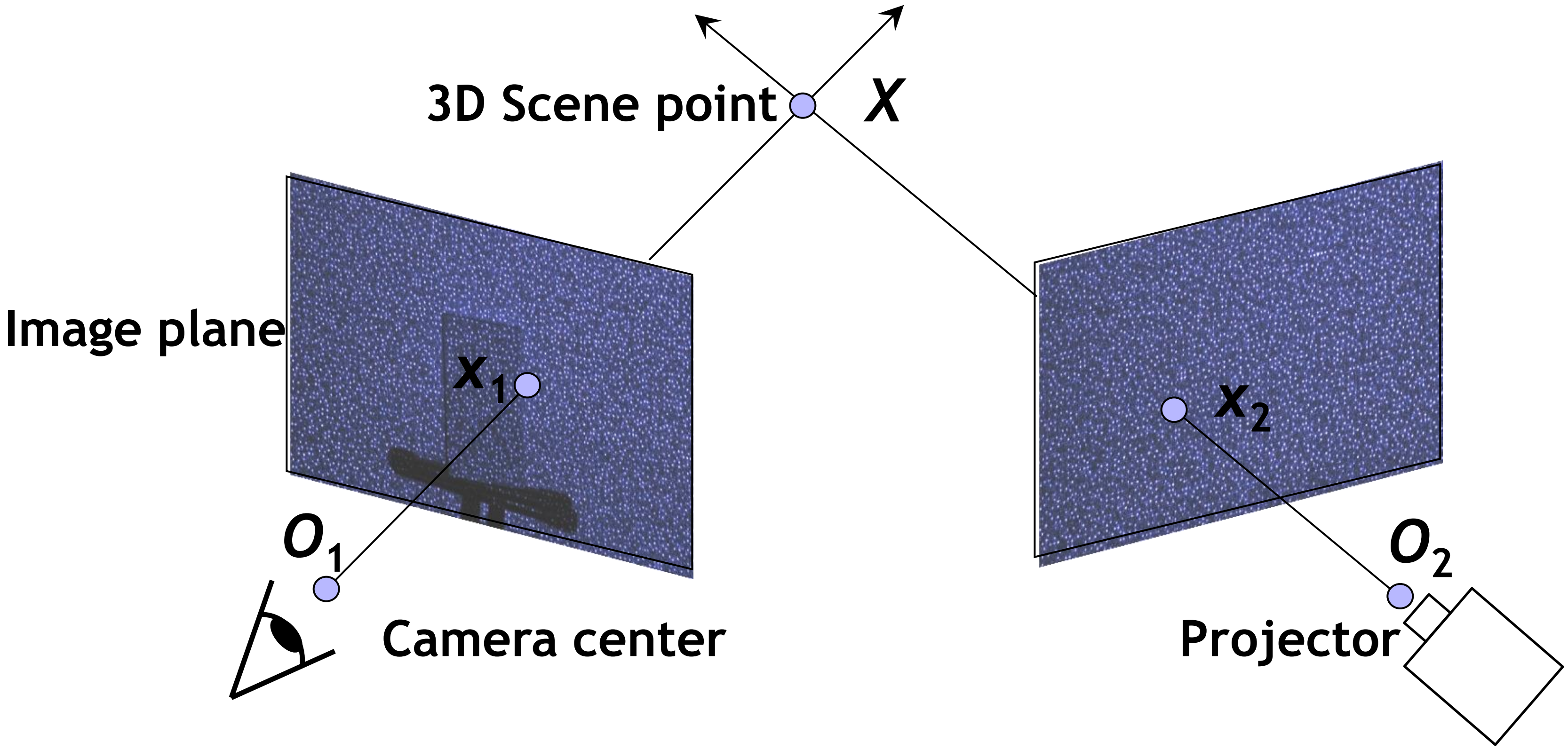


Recall: Optical Triangulation



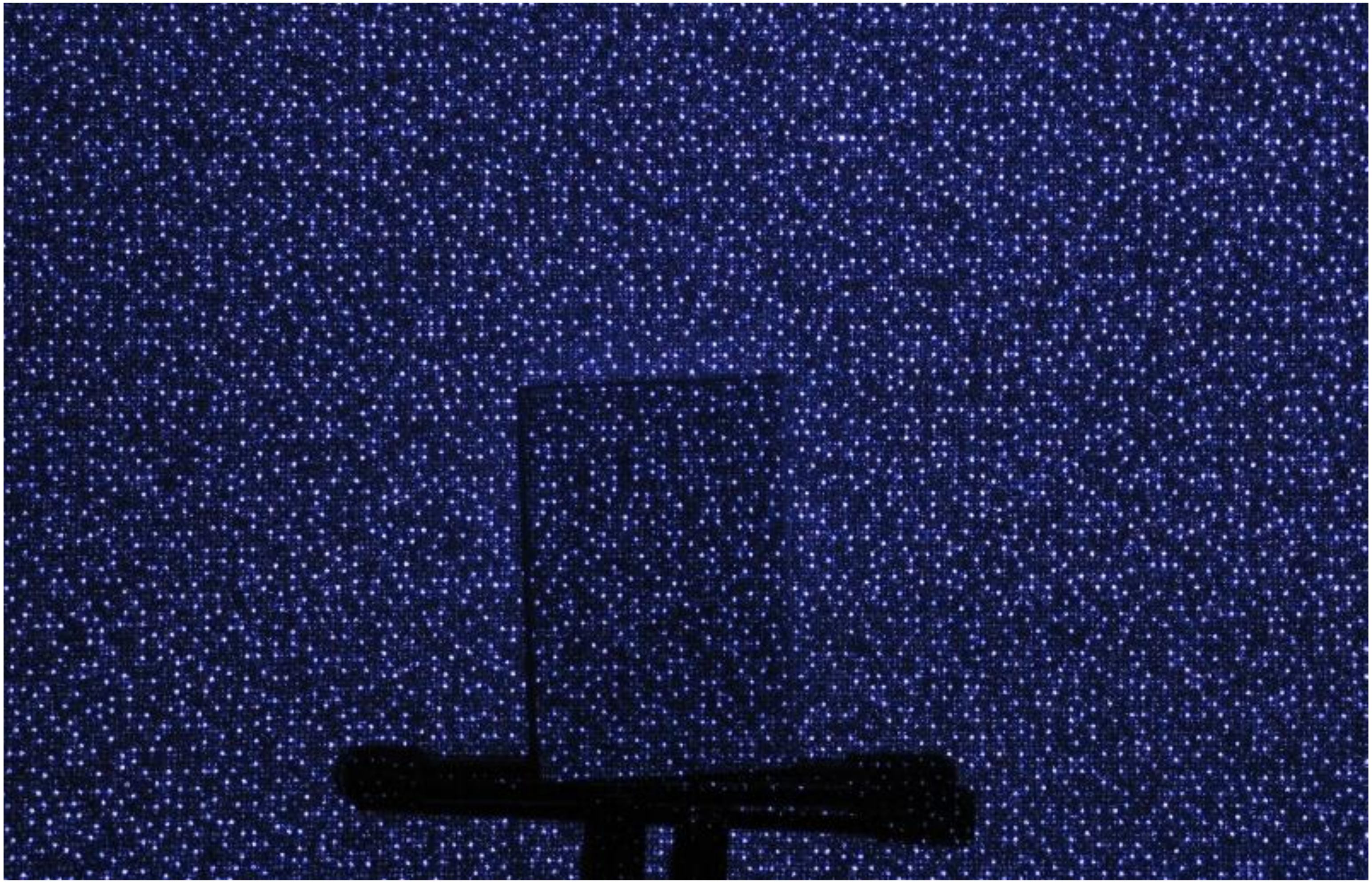
- Principle: 3D point given by intersection of two rays.
  - Crucial information: point correspondence
  - Most expensive and error-prone step in the pipeline…

Active Stereo with Structured Light



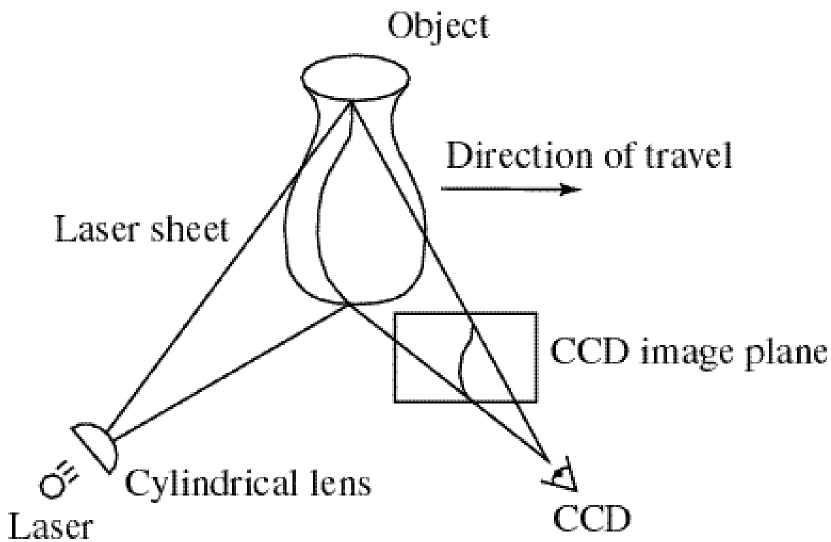
- **Idea: Replace one camera by a projector.**
  - Project "structured" light patterns onto the object
  - Simplifies the correspondence problem

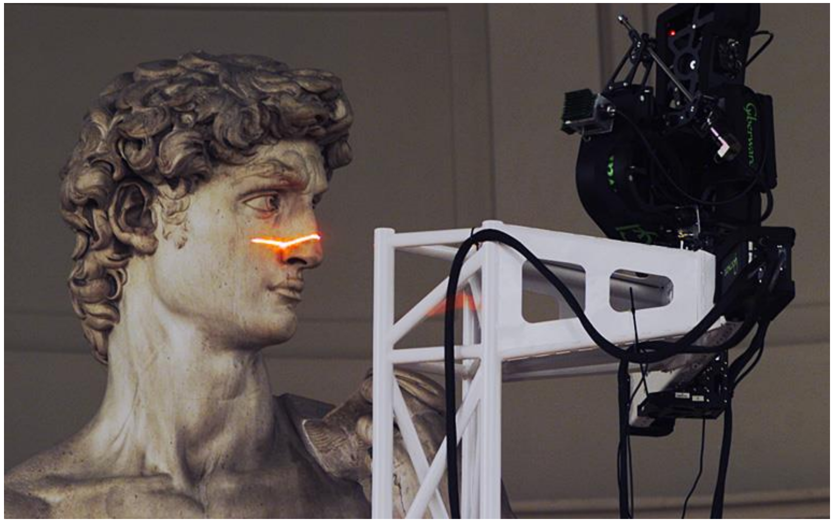
What the Kinect Sees…



3D Reconstruction with the Kinect

Laser Scanning





Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Laser Scanned Models

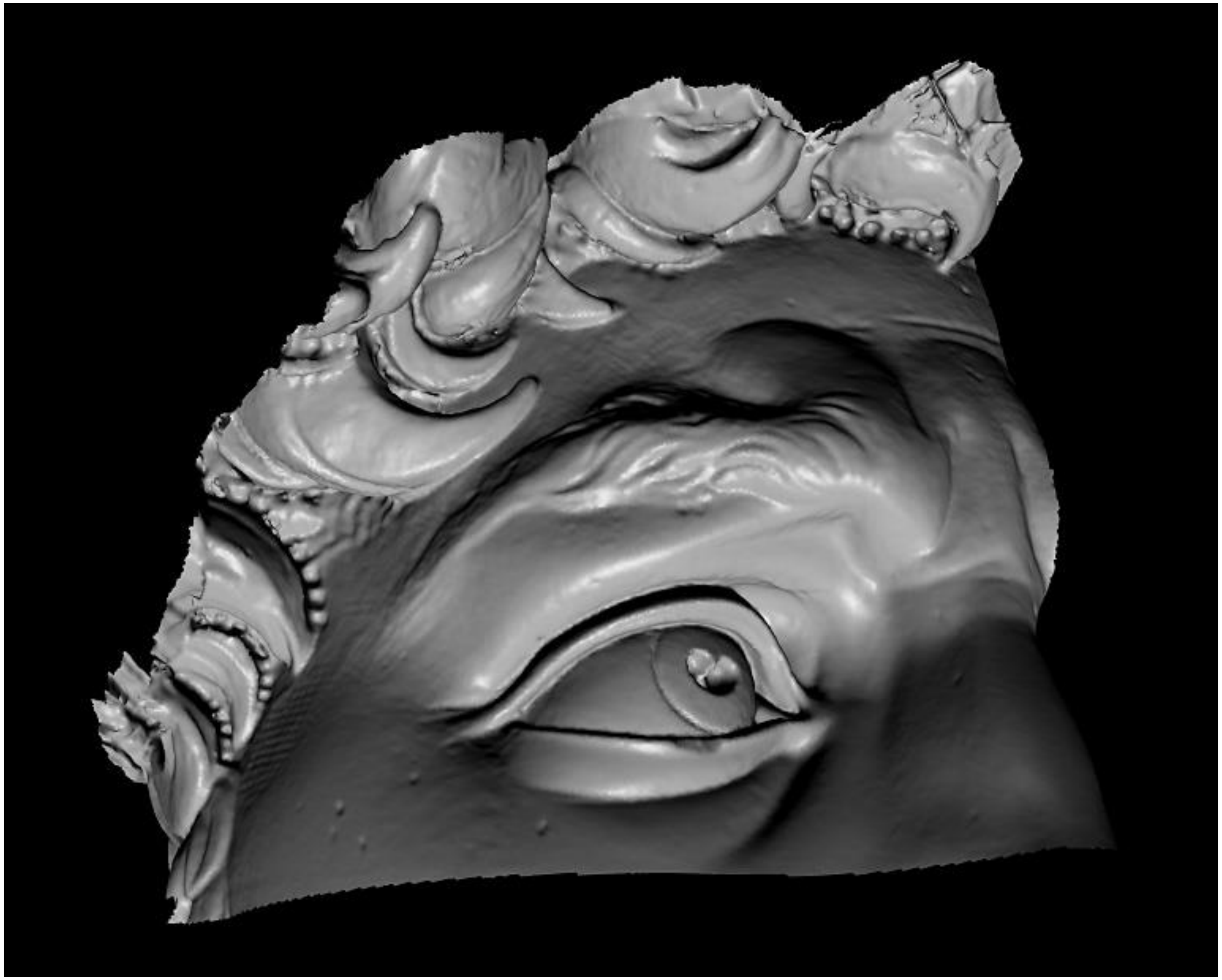
The Digital Michelangelo Project, Levoy et al.

Laser Scanned Models

The Digital Michelangelo Project, Levoy et al.

Slide credit: Steve Seitz

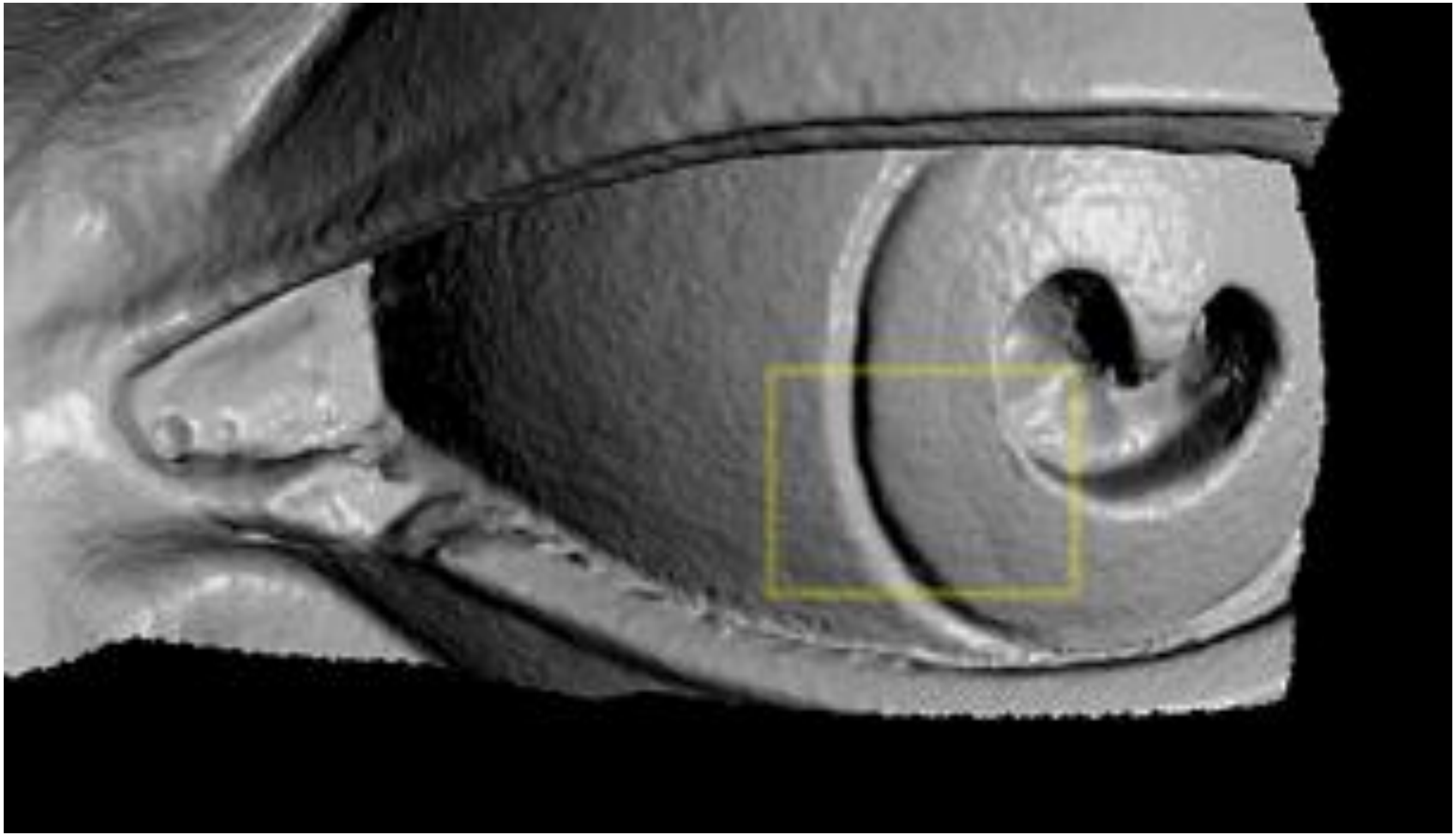
Laser Scanned Models



The Digital Michelangelo Project, Levoy et al.

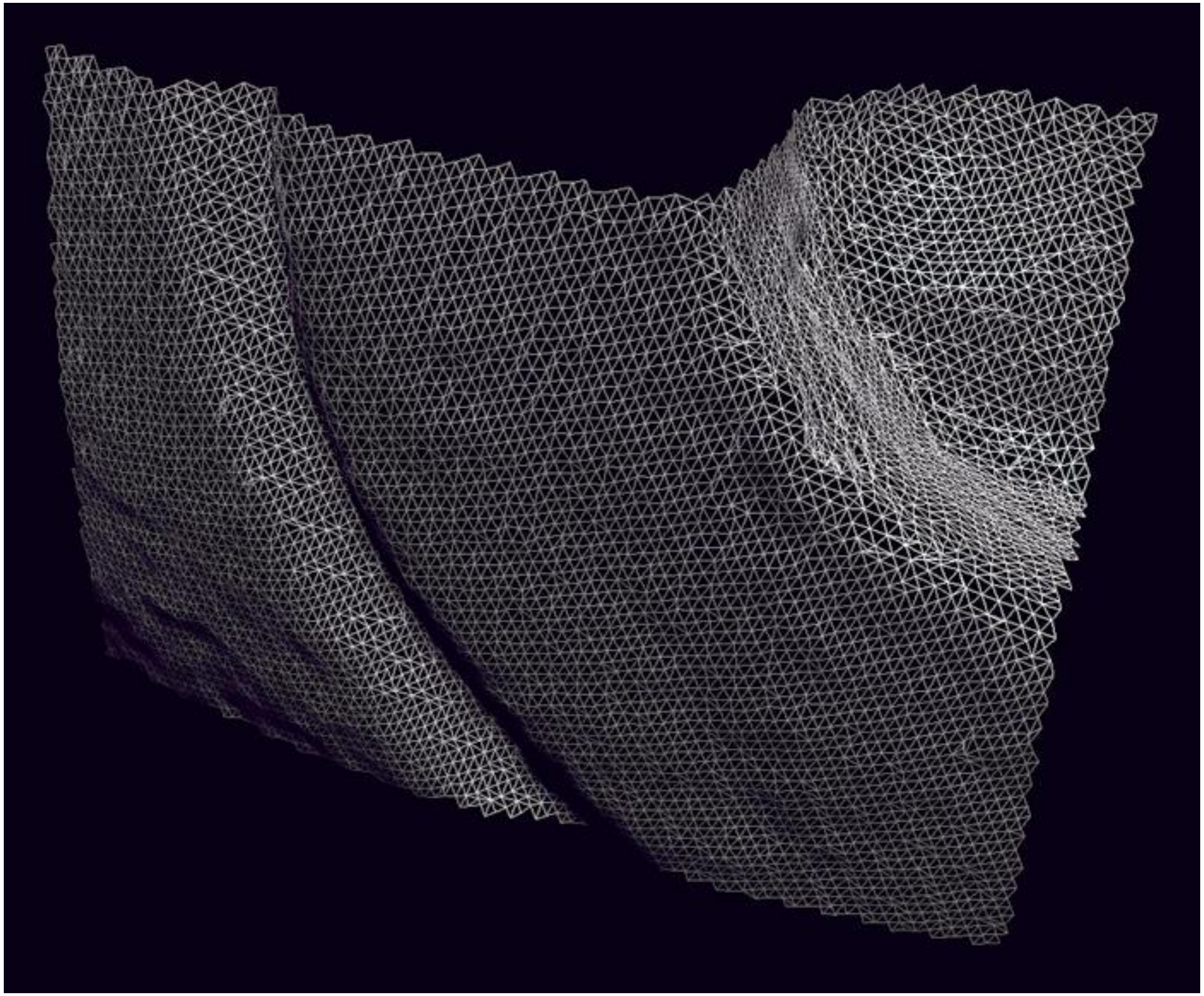
Slide credit: Steve Seitz

Laser Scanned Models



The Digital Michelangelo Project, Levoy et al.

Laser Scanned Models

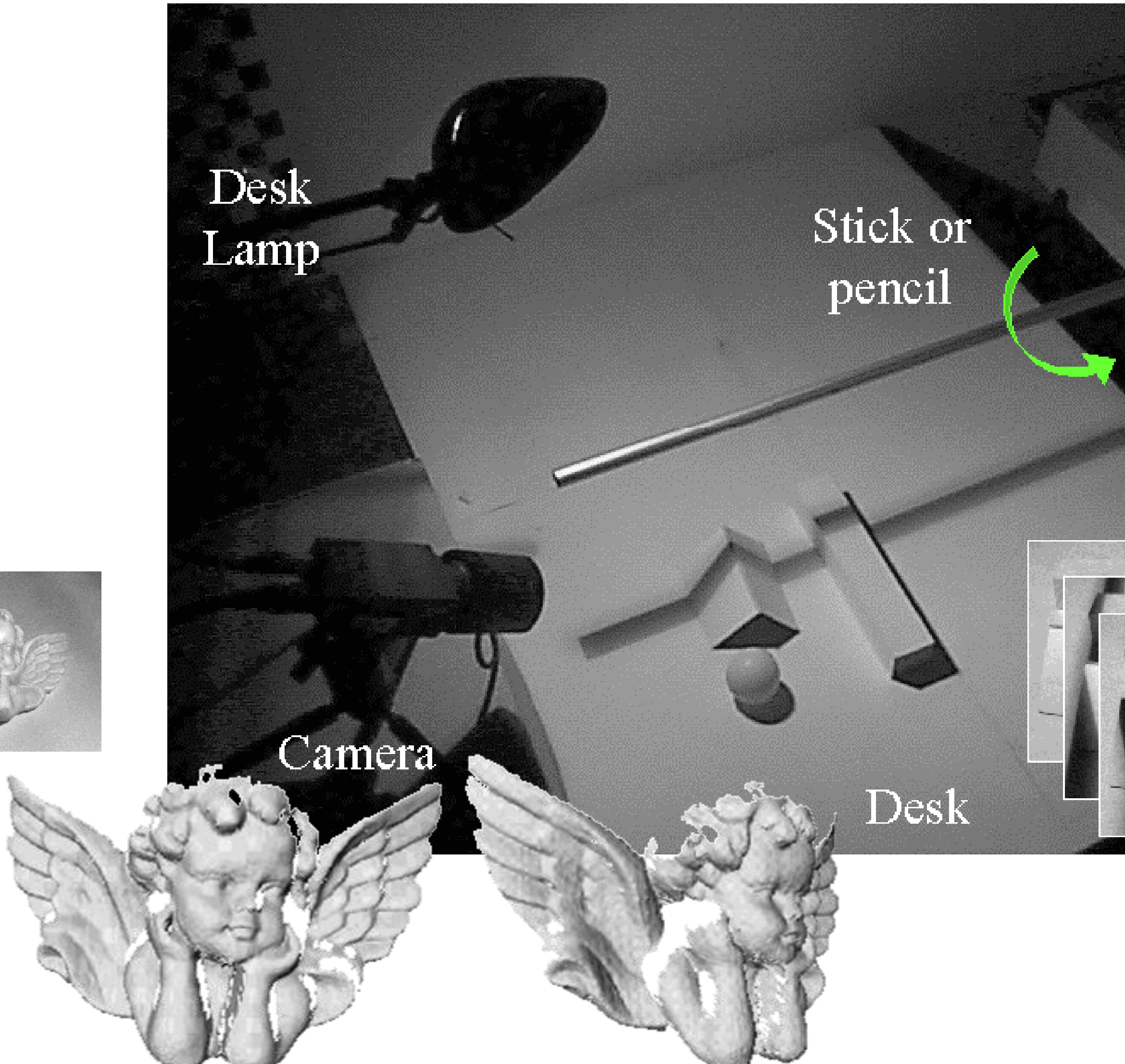


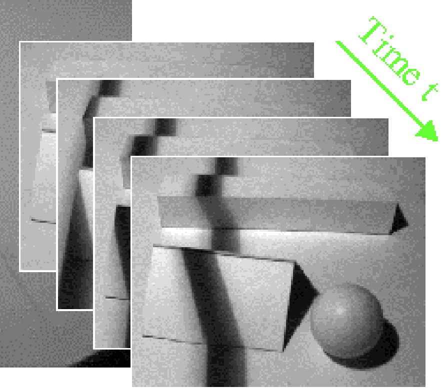
The Digital Michelangelo Project, **Levoy et al.**

Slide credit: Steve Seitz

Poor Man’s Scanner

The idea





Bouget and Perona, ICCV’98

Slightly More Elaborate (But Still Cheap)

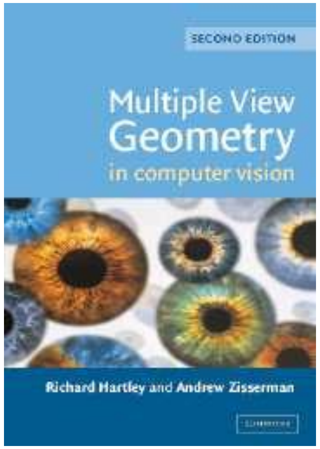
Software freely available from Robotics Institute TU Braunschweig

http://www.david-laserscanner.com/

References and Further Reading

- Background information on camera models and calibration algorithms can be found in Chapters 6 and 7 of

R. Hartley, A. Zisserman
Multiple View Geometry in Computer Vision
2nd Ed., Cambridge Univ. Press, 2004



- Also recommended: Chapter 9 of the same book on Epipolar geometry and the Fundamental Matrix and Chapter 11.1-11.6 on automatic computation of F.