

Computer Vision – Lecture 18

Repetition

09.07.2019

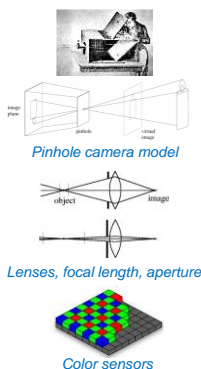
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 leibe@vision.rwth-aachen.de

Announcements

- Today, I'll summarize the most important points from the lecture.
 - It is an opportunity for you to ask questions...
 - ...or get additional explanations about certain topics.
 - So, please do ask.
- Today's slides are intended as an index for the lecture.
 - But they are not complete, won't be sufficient as only tool.
 - Also look at the exercises – they often explain algorithms in detail.

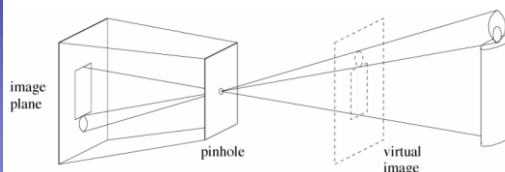
Repetition

- Image Processing Basics
 - Image Formation
 - Linear Filters
 - Edge & Structure Extraction
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
- 3D Reconstruction

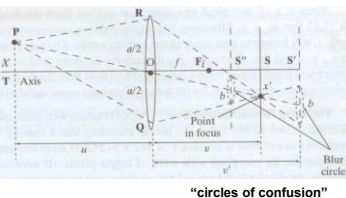


Recap: Pinhole Camera

- (Simple) standard and abstract model today
 - Box with a small hole in it
 - Works in practice



Recap: Focus and Depth of Field



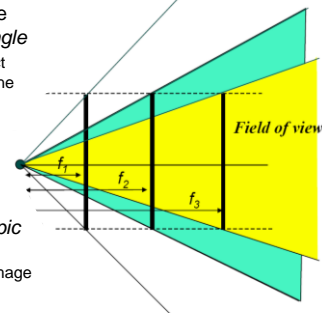
Thin lens: scene points at distinct depths come in focus at different image planes.

(Real camera lens systems have greater depth of field.)

- Depth of field: distance between image planes where blur is tolerable

Recap: Field of View and Focal Length

- As f gets smaller, image becomes more *wide angle*
 - More world points project onto the finite image plane
- As f gets larger, image becomes more *telescopic*
 - Smaller part of the world projects onto the finite image plane



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Recap: Color Sensing in Digital Cameras

Bayer grid

Estimate missing components from neighboring values (demosaicing)

Incoming Light
Filter Layer
Sensor Array
Resulting Pattern

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Source: Steve Seitz

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Gaussian Smoothing

Derivative operators

Gaussian/Laplacian pyramid

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Recap: Effect of Filtering

- Noise introduces high frequencies. To remove them, we want to apply a "low-pass" filter.
- The ideal filter shape in the frequency domain would be a box. But this transfers to a spatial sinc, which has infinite spatial support.
- A compact spatial box filter transfers to a frequency sinc, which creates artifacts.
- A Gaussian has compact support in both domains. This makes it a convenient choice for a low-pass filter.

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Recap: Gaussian Smoothing

see Exercise 1.1!

- Gaussian kernel

$$G_{\sigma} = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$
- Rotationally symmetric
- Weights nearby pixels more than distant ones
 - This makes sense as 'probabilistic' inference about the signal
- A Gaussian gives a good model of a fuzzy blob

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Image Source: Forsyth & Ponce

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Recap: Smoothing with a Gaussian

- Parameter σ is the "scale" / "width" / "spread" of the Gaussian kernel and controls the amount of smoothing.

```

for sigma=1:3:10
    h = fspecial('gaussian', fsize, sigma);
    out = imfilter(im, h);
    imshow(out);
    pause;
end
  
```

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Recap: Resampling with Prior Smoothing

| | | | | | |
|-----------|-----------|---------|---------|---------|-----------------------|
| 256 x 256 | 128 x 128 | 64 x 64 | 32 x 32 | 16 x 16 | |
| | | | | | Artifacts! |
| | | | | | no smoothing |
| | | | | | Gaussian $\sigma = 1$ |
| | | | | | Gaussian $\sigma = 2$ |

- Note: We cannot recover the high frequencies, but we can avoid artifacts by smoothing before resampling.

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Image Source: Forsyth & Ponce

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Recap: The Gaussian Pyramid

Low resolution

High resolution

$G_4 = (G_3 * \text{gaussian}) \downarrow 2$

$G_3 = (G_2 * \text{gaussian}) \downarrow 2$

$G_2 = (G_1 * \text{gaussian}) \downarrow 2$

$G_1 = (G_0 * \text{gaussian}) \downarrow 2$

$G_0 = \text{Image}$

blur

down-sample

blur

down-sample

blur

down-sample

blur

down-sample

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Source: Irani & Bar

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Recap: Derivatives and Edges...

1st derivative

2nd derivative

Maxima of first derivative

"zero crossings" of second derivative

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Recap: 2D Edge Detection Filters

see Exercise 1.3!

Gaussian

$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

Derivative of Gaussian

$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

Laplacian of Gaussian

$$\nabla^2 h_{\sigma}(u, v)$$

∇^2 is the Laplacian operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

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Canny edge detector

Hough transform for lines

Hough transform for circles

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Recap: Canny Edge Detector

see Exercise 1.4!

- Filter image with derivative of Gaussian
- Find magnitude and orientation of gradient
- Non-maximum suppression:
 - Thin multi-pixel wide "ridges" down to single pixel width
- Linking and thresholding (hysteresis):
 - Define two thresholds: low and high
 - Use the high threshold to start edge curves and the low threshold to continue them

MATLAB:

```
>> edge(image, 'canny');
>> help edge
```

adapted from D. Lowe, L. Fei-Fei

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Recap: Edges vs. Boundaries

Edges useful signal to indicate occluding boundaries, shape.

Here the raw edge output is not so bad...

...but quite often boundaries of interest are fragmented, and we have extra "clutter" edge points.

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Recap: Fitting and Hough Transform

Given a model of interest, we can overcome some of the missing and noisy edges using fitting techniques.

With voting methods like the Hough transform, detected points vote on possible model parameters.

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Recap: Hough Transform

Image space Hough (parameter) space

- How can we use this to find the most likely parameters (m, b) for the most prominent line in the image space?
 - Let each edge point in image space vote for a set of possible parameters in Hough space
 - Accumulate votes in discrete set of bins; parameters with the most votes indicate line in image space.

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Recap: Hough Transf. Polar Parametrization

- Usual (m, b) parameter space problematic: can take on infinite values, undefined for vertical lines.

d : perpendicular distance from line to origin
 θ : angle the perpendicular makes with the x-axis

$x \cos \theta - y \sin \theta = d$

- Point in image space \Rightarrow sinusoid segment in Hough space

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Recap: Hough Transform for Circles

- Circle: center (a, b) and radius r
 $(x_i - a)^2 + (y_i - b)^2 = r^2$
- For an unknown radius r , unknown gradient direction

Image space Hough space

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Recap: Generalized Hough Transform

- What if want to detect arbitrary shapes defined by boundary points and a reference point?

At each boundary point, compute displacement vector:
 $r = a - p_i$

For a given model shape: store these vectors in a table indexed by gradient orientation θ .

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D.H. Ballard, Generalizing the Hough Transform to Detect Arbitrary Shapes, 1980.
 Slide credit: Kristen Grauman

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Repetition

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 - Segmentation and Grouping
 - Segmentation as Energy Minimization
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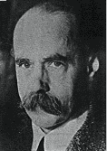
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Recap: Gestalt Theory

- Gestalt: whole or group
 - Whole is greater than sum of its parts
 - Relationships among parts can yield new properties/features
- Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

"I stand at the window and see a house, trees, sky. Theoretically I might say there were 327 brightnesses and nuances of colour. Do I have '327'? No. I have sky, house, and trees."

Max Wertheimer (1880-1943)

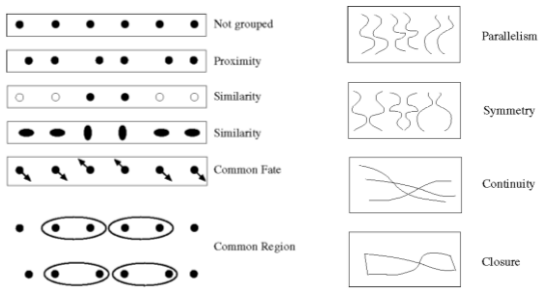


Untersuchungen zur Lehre von der Gestalt, *Psychologische Forschung*, Vol. 4, pp. 301-350, 1923
<http://psy.ed.asu.edu/~classics/Wertheimer/Forms/forms.htm>

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Recap: Gestalt Factors



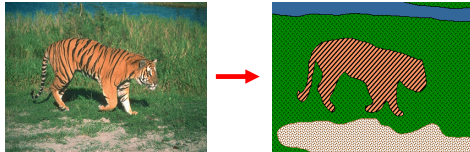
- These factors make intuitive sense, but are very difficult to translate into algorithms.

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Recap: Image Segmentation

- Goal: identify groups of pixels that go together



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Recap: K-Means Clustering

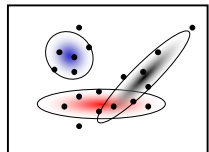
- Basic idea: randomly initialize the k cluster centers, and iterate between the two following steps
 - Randomly initialize the cluster centers, c_1, \dots, c_k
 - Given cluster centers, determine points in each cluster
 - For each point p , find the closest c_i . Put p into cluster i
 - Given points in each cluster, solve for c_i
 - Set c_i to be the mean of points in cluster i
 - If c_i have changed, repeat Step 2
- Properties
 - Will always converge to *some* solution
 - Can be a "local minimum"
 - Does not always find the global minimum of objective function:

$$\sum_{\text{clusters } i} \sum_{\text{points } p \text{ in cluster } i} \|p - c_i\|^2$$

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Recap: Expectation Maximization (EM)



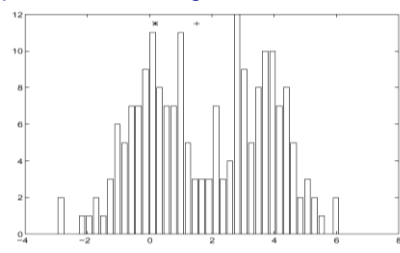
- Goal
 - Find blob parameters θ that maximize the likelihood function:
- Approach:

$$p(\text{data}|\theta) = \prod_{n=1}^N p(x_n|\theta)$$
 - E-step: given current guess of blobs, compute ownership of each point
 - M-step: given ownership probabilities, update blobs to maximize likelihood function
 - Repeat until convergence

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Recap: Mean-Shift Algorithm



- Iterative Mode Search
 - Initialize random seed, and window W
 - Calculate center of gravity (the "mean") of W : $\sum_{x \in W} xH(x)$
 - Shift the search window to the mean
 - Repeat Step 2 until convergence

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Recap: Mean-Shift Clustering

- Cluster: all data points in the attraction basin of a mode
- Attraction basin: the region for which all trajectories lead to the same mode

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Slide by Y. Ukrainitz & B. Sarel

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see Exercise 2.1!

Recap: Mean-Shift Segmentation

- Find features (color, gradients, texture, etc)
- Initialize windows at individual pixel locations
- Perform mean shift for each window until convergence
- Merge windows that end up near the same "peak" or mode

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Slide credit: Svetlana Lazebnik

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Repetition

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Recap: MRFs for Image Segmentation

- MRF formulation

Unary potentials $\phi(x_i, y_i)$

Pairwise potentials $\psi(x_i, x_j)$

\Rightarrow Minimize the energy

$$E(\mathbf{x}, \mathbf{y}) = \sum_i \phi(x_i, y_i) + \sum_{i,j} \psi(x_i, x_j)$$

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Slide adapted from Phil Torr

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Recap: Energy Formulation

- Energy function

$$E(\mathbf{x}, \mathbf{y}) = \underbrace{\sum_i \phi(x_i, y_i)}_{\text{Unary potentials}} + \underbrace{\sum_{i,j} \psi(x_i, x_j)}_{\text{Pairwise potentials}}$$

- Unary potentials ϕ
 - Encode local information about the given pixel/patch
 - How likely is a pixel/patch to belong to a certain class (e.g. foreground/background)?
- Pairwise potentials ψ
 - Encode neighborhood information
 - How different is a pixel/patch's label from that of its neighbor? (e.g. based on intensity/color/texture difference, edges)

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Recap: How to Set the Potentials?

- Unary potentials
 - E.g. color model, modeled with a Mixture of Gaussians

$$\phi(x_i, y_i; \theta_\phi) = \log \sum_k \theta_\phi(x_i, k) p(k|x_i) \mathcal{N}(y_i; \bar{y}_k, \Sigma_k)$$

\Rightarrow Learn color distributions for each label

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Recap: How to Set the Potentials?

- Pairwise potentials
 - Potts Model
 - $\psi(x_i, x_j; \theta_\psi) = \theta_\psi \delta(x_i \neq x_j)$
 - Simplest discontinuity preserving model.
 - Discontinuities between any pair of labels are penalized equally.
 - Useful when labels are unordered or number of labels is small.
 - Extension: "Contrast sensitive Potts model"
 - $\psi(x_i, x_j, g_{ij}(y); \theta_\psi) = \theta_\psi g_{ij}(y) \delta(x_i \neq x_j)$
 - where
 - $g_{ij}(y) = e^{-\beta \|y_i - y_j\|^2}$ $\beta = 2 / \text{avg}(\|y_i - y_j\|^2)$
 - ⇒ Discourages label changes except in places where there is also a large change in the observations.

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Recap: Graph-Cuts Energy Minimization

see Exercise 2.2!

- Solve an equivalent graph cut problem
 - Introduce extra nodes: source and sink
 - Weight connections to source/sink (t-links) by $\phi(x_i = s)$ and $\phi(x_i = t)$, respectively.
 - Weight connections between nodes (n-links) by $\psi(x_i, x_j)$.
 - Find the minimum cost cut that separates source from sink.
 - ⇒ Solution is equivalent to minimum of the energy.
- s-t Mincut can be solved efficiently
 - Dual to the well-known max flow problem
 - Very efficient algorithms available for regular grid graphs (1-2 MPixels/s)
 - Globally optimal result for 2-class problems

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Recap: When Can s-t Graph Cuts Be Applied?

$$E(L) = \sum_p E_p(L_p) + \sum_{p,q \in N} E(L_p, L_q)$$

Unary potentials Pairwise potentials
t-links n-links $L_p \in \{s, t\}$

- s-t graph cuts can only globally minimize binary energies that are **submodular**. [Boros & Hummer, 2002; Kolmogorov & Zabih, 2004]

$E(L)$ can be minimized by s-t graph cuts $\iff E(s,s) + E(t,t) \leq E(s,t) + E(t,s)$

Submodularity ("convexity")

- Submodularity is the discrete equivalent to convexity.
 - ⇒ Implies that every local energy minimum is a global minimum.
 - ⇒ Solution will be globally optimal.

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First Applications Take Up Shape...

- Line detection
- Circle detection
- Binary Segmentation
- Skin color detection
- Simple shape recognition

Image recognition

Forget it!

Image Source: <http://www.flickr.com/photos/janelst/2805412867/>

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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
 - Sliding Window based Object Detection
- Local Features & Matching
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Sliding window principle

Boosting SVM

HOG detector Train cascade of classifiers with AdaBoost Viola-Jones face detector

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Recap: Sliding-Window Object Detection

- If object may be in a cluttered scene, slide a window around looking for it.

Car/non-car Classifier

- Essentially, this is a brute-force approach with many local decisions.

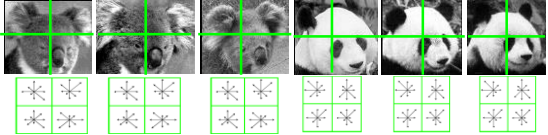
Slide credit: Kristen Grauman

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Recap: Gradient-based Representations

- Consider edges, contours, and (oriented) intensity gradients



- Summarize local distribution of gradients with histogram
 - Locally orderless: offers invariance to small shifts and rotations
 - Contrast-normalization: try to correct for variable illumination

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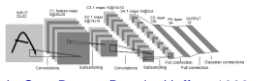
Classifier Construction: Many Choices...

Nearest Neighbor




Berg, Berg, Malik 2005,
Chum, Zisserman 2007,
Boiman, Shechtman, Irani 2008, ...

Neural networks



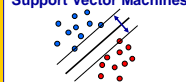
LeCun, Bottou, Bengio, Haffner 1998
Rowley, Baluja, Kanade 1998
...

Boosting




Viola, Jones 2001,
Torralba et al. 2004,
Opelt et al. 2006,
Benenson 2012, ...

Support Vector Machines



Vapnik, Schölkopf 1995,
Papageorgiou, Poggio '01,
Dalal, Triggs 2005,
Vedaldi, Zisserman 2012

Randomized Forests



Amit, Geman 1997,
Breiman 2001,
Lepetit, Fua 2006,
Gall, Lempitsky 2009, ...

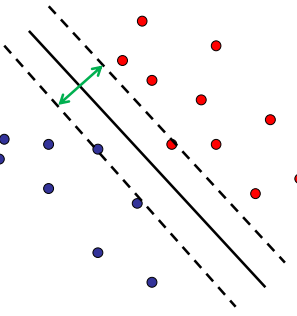
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Recap: Support Vector Machines (SVMs)



- Discriminative classifier based on *optimal separating hyperplane* (i.e. line for 2D case)
- Maximize the *margin* between the positive and negative training examples

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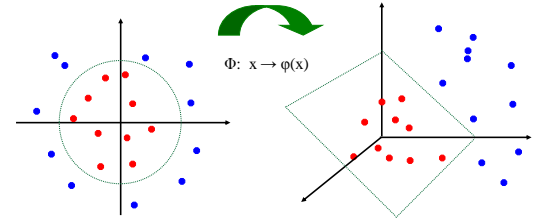
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Recap: Non-Linear SVMs

- General idea: The original input space can be mapped to some higher-dimensional feature space where the training set is separable:



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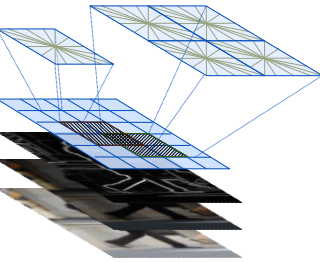
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Slide from Andrew Moore's tutorial: <http://www.autonlab.org/tutorials/svm.html>

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Recap: HOG Descriptor Processing Chain

- SVM Classification
 - Typically using a linear SVM



Object/Non-object

Linear SVM

Collect HOGs over detection window

Contrast normalize over overlapping spatial cells

Weighted vote in spatial & orientation cells

Compute gradients

Gamma compression

Image Window

Goal

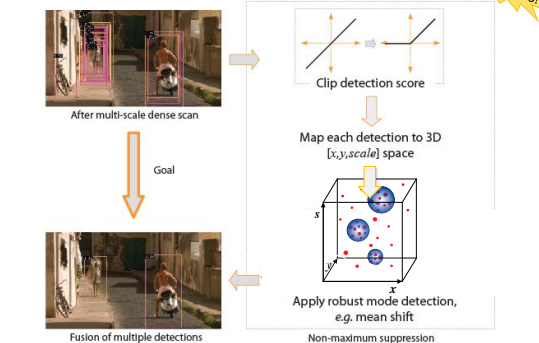
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Slide adapted from Navneet Dalal

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Recap: Non-Maximum Suppression



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Image source: Navneet Dalal, PhD Thesis
see Exercise 2.3!

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Classifier Construction: Many Choices...

Nearest Neighbor

Shakhnarovich, Viola, Darrell 2003
Berg, Berg, Malik 2005,
Boiman, Shechtman, Irani 2008, ...

Neural networks

LeCun, Bottou, Bengio, Haffner 1998
Rowley, Baluja, Kanade 1998
...

Boosting

Viola, Jones 2001,
Torralba et al. 2004,
Opelt et al. 2006,
Benenson 2012, ...

Support Vector Machines

Vapnik, Schölkopf 1995,
Papageorgiou, Poggio '01,
Dalal, Triggs 2005,
Vedaldi, Zisserman 2012

Randomized Forests

Amit, Geman 1997,
Breiman 2001,
Lepetit, Fua 2006,
Gall, Lempitsky 2009, ...

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Recap: AdaBoost

Final classifier is combination of the weak classifiers

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Recap: Viola-Jones Face Detection

"Rectangular" filters

Feature output is difference between adjacent regions

Value at (x,y) is sum of pixels above and to the left of (x,y)

$$D = 1 + 4 - (2 + 3) = A + (A + B + C + D) - (A + C + A + B) = D$$

Efficiently computable with integral image: any sum can be computed in constant time

Avoid scaling images → scale features directly for same cost

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Computer Vision Summer'19 | Slide credit: Kristen Grauman | B. Leibe | [Viola & Jones, CVPR 2001]

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Recap: AdaBoost Feature+Classifier Selection

- Want to select the single rectangle feature and threshold that best separates **positive** (faces) and **negative** (non-faces) training examples, in terms of *weighted* error.

Resulting weak classifier:

$$h_t(x) = \begin{cases} +1 & \text{if } f_t(x) > \theta_t \\ -1 & \text{otherwise} \end{cases}$$

For next round, reweight the examples according to errors, choose another filter/threshold combo.

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Computer Vision Summer'19 | Slide credit: Kristen Grauman | B. Leibe | [Viola & Jones, CVPR 2001]

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Recap: Viola-Jones Face Detector

- Train with 5K positives, 350M negatives
- Real-time detector using 38 layer cascade
- 6061 features in final layer
- [Implementation available in OpenCV: <http://sourceforge.net/projects/opencvlibrary/>]

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 - Local Features – Detection and Description
 - Recognition with Local Features
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Recap: Local Feature Matching Pipeline

1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Similarity measure $d(f_A, f_B) < T$

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Recap: Requirements for Local Features

- Problem 1:
 - Detect the same point *independently* in both images
- Problem 2:
 - For each point correctly recognize the corresponding one

We need a repeatable detector!

We need a reliable and distinctive descriptor!

Slide credit: Darya Erolova, Denis Simakov B. Leibe 56

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Recap: Harris Detector [Harris88]

see Exercise 3.2!

- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

1. Image derivatives
2. Square of derivatives
3. Gaussian filter $g(\sigma)$

4. Cornerness function – two strong eigenvalues

$$R = \det[M(\sigma_I, \sigma_D)] - \alpha[\text{trace}(M(\sigma_I, \sigma_D))]^2$$

$$= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

5. Perform non-maximum suppression

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Recap: Harris Detector Responses

[Harris88]

Effect: A very precise corner detector.

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Recap: Hessian Detector

[Beaudet78] see Exercise 3.2!

- Hessian determinant

$$\text{Hessian}(I) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix}$$

$$\det(\text{Hessian}(I)) = I_{xx} I_{yy} - I_{xy}^2$$

In Matlab:

$$I_{xx} * I_{yy} - (I_{xy})^2$$

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Hessian Detector – Responses

[Beaudet78]

Effect: Responses mainly on corners and strongly textured areas.

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Recap: Automatic Scale Selection

- Function responses for increasing scale (scale signature)

$f(I_{k,j}(x, \sigma))$

$f(I_{k,j}(x', \sigma))$

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Recap: Laplacian-of-Gaussian (LoG)

- Interest points:
 - Local maxima in scale space of Laplacian-of-Gaussian

$L_{xx}(\sigma) + L_{yy}(\sigma)$

Scale

\Rightarrow List of (x, y, σ)

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Recap: LoG Detector Responses

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Slide credit: Svetlana Lazebnik B. Leibe

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Recap: Key point localization with DoG

- Efficient implementation
 - Approximate LoG with a difference of Gaussians (DoG)
- Approach DoG Detector
 - Detect maxima of difference-of-Gaussian in scale space
 - Reject points with low contrast (threshold)
 - Eliminate edge responses

Candidate keypoints:
list of (x, y, σ)

Image source: David Lowe

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[Mikolajczyk '01]

Recap: Harris-Laplace

- Initialization: Multiscale Harris corner detection
- Scale selection based on Laplacian (same procedure with Hessian \Rightarrow Hessian-Laplace)

Harris points

Harris-Laplace points

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Recap: Orientation Normalization

- Compute orientation histogram [Lowe, SIFT, 1999]
- Select dominant orientation
- Normalize: rotate to fixed orientation

0 \uparrow 2π

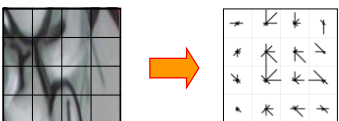
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Slide adapted from David Lowe B. Leibe

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Recap: SIFT Feature Descriptor

- Scale Invariant Feature Transform
- Descriptor computation:
 - Divide patch into 4x4 sub-patches: 16 cells
 - Compute histogram of gradient orientations (8 reference angles) for all pixels inside each sub-patch
 - Resulting descriptor: 4x4x8 = 128 dimensions



D.G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

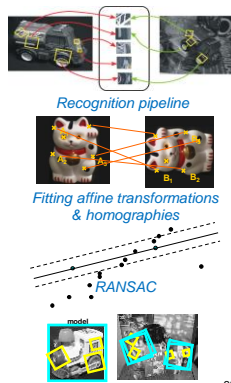
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Slide credit: Svetlana Lazebnik B. Leibe

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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
 - Local Features – Detection and Description
 - Recognition with Local Features
- Deep Learning
- 3D Reconstruction



Recognition pipeline

Fitting affine transformations & homographies

RANSAC

Gen. Hough Transform

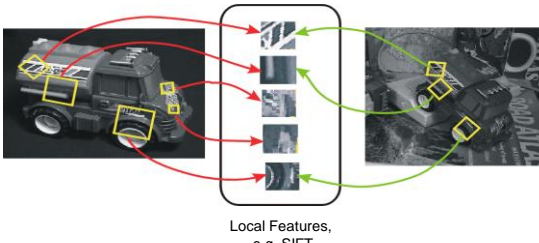
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Recap: Recognition with Local Features

- Image content is transformed into local features that are invariant to translation, rotation, and scale
- Goal: Verify if they belong to a consistent configuration



Local Features, e.g. SIFT

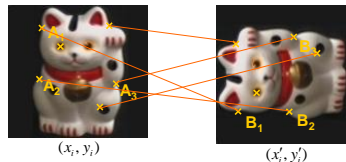
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Slide credit: David Lowe B. Leibe

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Recap: Fitting an Affine Transformation

- Assuming we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\begin{bmatrix} x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix}$$

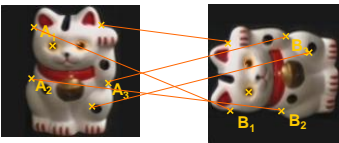
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Recap: Fitting a Homography

- Estimating the transformation



Homogenous coordinates

Image coordinates

Matrix notation $x' = Hx$

$$x'' = \frac{1}{z'} x'$$

$$x''_A = \frac{h_{11}x_B + h_{12}y_B + h_{13}}{h_{31}x_B + h_{32}y_B + 1}$$

$$y''_A = \frac{h_{21}x_B + h_{22}y_B + h_{23}}{h_{31}x_B + h_{32}y_B + 1}$$

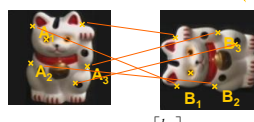
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Recap: Fitting a Homography

- Estimating the transformation



$$h_{11}x_B + h_{12}y_B + h_{13} - x_A h_{31} - x_A h_{32}y_B - x_A = 0$$

$$h_{21}x_B + h_{22}y_B + h_{23} - y_A h_{31} - y_A h_{32}y_B - y_A = 0$$

$$\begin{bmatrix} x_B & y_B & 1 & 0 & 0 & 0 & -x_A & -x_A y_B & -x_A \\ 0 & 0 & 0 & x_B & y_B & 1 & -y_A & -y_A y_B & -y_A \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{B_1} & y_{B_1} & 1 & 0 & 0 & 0 & -x_{A_1} & -x_{A_1} y_{B_1} & -x_{A_1} \\ x_{B_2} & y_{B_2} & 1 & 0 & 0 & 0 & -x_{A_2} & -x_{A_2} y_{B_2} & -x_{A_2} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ x_{B_3} & y_{B_3} & 1 & 0 & 0 & 0 & -x_{A_3} & -x_{A_3} y_{B_3} & -x_{A_3} \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}$$

$Ah = 0$

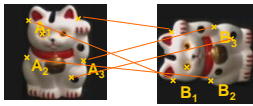
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Recap: Fitting a Homography

- Estimating the transformation
- Solution:
 - Null-space vector of A
 - Corresponds to smallest eigenvector



$$Ah = 0$$

SVD

$$A = UDV^T = U \begin{bmatrix} d_{11} & \dots & d_{19} \\ \vdots & \ddots & \vdots \\ d_{91} & \dots & d_{99} \end{bmatrix} \begin{bmatrix} v_{11} & \dots & v_{19} \\ \vdots & \ddots & \vdots \\ v_{91} & \dots & v_{99} \end{bmatrix}^T$$

$$h = \begin{bmatrix} v_{19} & \dots & v_{99} \end{bmatrix} \text{ Minimizes least square error}$$

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Recap: RANSAC

see Exercise 5.1!

RANSAC loop:

1. Randomly select a *seed group* of points on which to base transformation estimate (e.g., a group of matches)
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers


- Keep the transformation with the largest number of inliers

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Recap: RANSAC Line Fitting Example

- Task: Estimate the best line

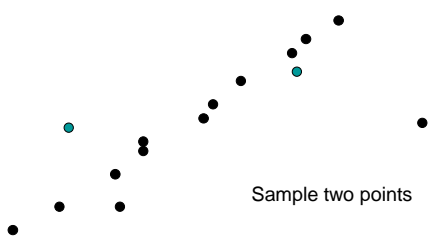


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Recap: RANSAC Line Fitting Example

- Task: Estimate the best line

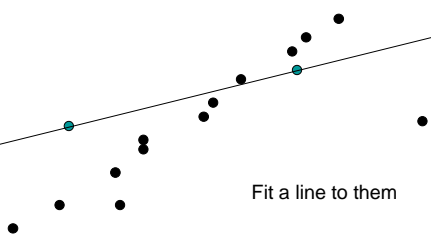


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Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



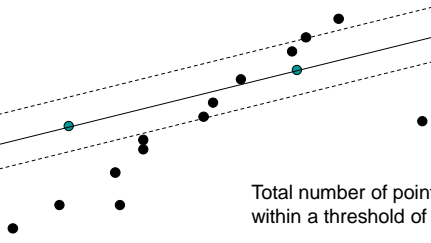
Fit a line to them

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Recap: RANSAC Line Fitting Example

- Task: Estimate the best line



Total number of points within a threshold of line.

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Recap: RANSAC Line Fitting Example

- Task: Estimate the best line

Repeat, until we get a good result.

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Recap: Feature Matching Example

- Find best stereo match within a square search window (here 300 pixels²)
- Global transformation model: epipolar geometry

before RANSAC

after RANSAC

Images from Hartley & Zisserman

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Recap: Generalized Hough Transform

- Suppose our features are scale- and rotation-invariant
 - Then a single feature match provides an alignment hypothesis (translation, scale, orientation).
 - Of course, a hypothesis from a single match is unreliable.
 - Solution: let each match vote for its hypothesis in a Hough space with very coarse bins.

model

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Application: Panorama Stitching

<http://www.cs.ubc.ca/~mbrown/autostitch/autostitch.html>

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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
 - Convolutional Neural Networks (CNNs)
 - Deep Learning Background
 - CNNs for Object Detection
 - CNNs for Semantic Segmentation
 - CNNs for Matching & RNNs
- 3D Reconstruction

Convolutional Neural Networks

Convolution layers

| | | | |
|---|---|---|---|
| 1 | 5 | 2 | 7 |
| 8 | 8 | 7 | 8 |
| 3 | 7 | 1 | 0 |
| 1 | 2 | 3 | 7 |

Pooling layers

AlexNet, VGGNet, GoogLeNet, ResNet

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Recap: Convolutional Neural Networks

see Exercise 4.1!

INPUT 32x32

“LeNet” architecture

Convolutions

Subsampling

Convolutions

Subsampling

Full connection

Full connection

Gaussian connections

- Neural network with specialized connectivity structure
 - Stack multiple stages of feature extractors
 - Higher stages compute more global, more invariant features
 - Classification layer at the end

Y. LeCun, L. Bottou, Y. Bengio, and P. Haffner, [Gradient-based learning applied to document recognition](#), Proceedings of the IEEE 86(11): 2278–2324, 1998.

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Recap: CNN Structure

- Feed-forward feature extraction
 1. Convolve input with learned filters
 2. Non-linearity
 3. Spatial pooling
 4. (Normalization)
- Supervised training of convolutional filters by back-propagating classification error

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Slide credit: Svetlana Lazebnik B. Leibe

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Recap: Intuition of CNNs

- Convolutional network
 - Share the same parameters across different locations
 - Convolutions with learned kernels
- Learn *multiple* filters
 - E.g. 1000×1000 image
 - 100 filters
 - 10×10 filter size
 - ⇒ only 10k parameters
- Result: Response map
 - size: 1000×1000×100
 - Only memory, not params!

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Slide adapted from Marc'Aurelio Ranzato B. Leibe Image source: Yann LeCun

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Recap: Convolution Layers

Naming convention:

- All Neural Net activations arranged in 3 dimensions
 - Multiple neurons all looking at the same input region, stacked in depth
 - Form a single $[1 \times 1 \times \text{depth}]$ depth column in output volume.

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Slide credit: FeiFei Li, Andrei Karpathy B. Leibe

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Recap: Activation Maps

Activations: one filter = one depth slice (or activation map)

5×5 filters

Each activation map is a depth slice through the output volume.

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Slide adapted from FeiFei Li, Andrei Karpathy B. Leibe

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Recap: Pooling Layers

Single depth slice

| | | | |
|---|---|---|---|
| 1 | 1 | 2 | 4 |
| 5 | 6 | 7 | 8 |
| 3 | 2 | 1 | 0 |
| 1 | 2 | 3 | 4 |

max pool with 2x2 filters and stride 2

| | |
|---|---|
| 6 | 8 |
| 3 | 4 |

- Effect:
 - Make the representation smaller without losing too much information
 - Achieve robustness to translations

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Slide adapted from FeiFei Li, Andrei Karpathy B. Leibe

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Recap: Effect of Multiple Convolution Layers

Low-Level Feature Mid-Level Feature High-Level Feature Trainable Classifier

Feature visualization of convolutional net trained on ImageNet from [Zeiler & Fergus 2013]

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Slide credit: Yann LeCun B. Leibe

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Recap: AlexNet (2012)

• Similar framework as LeNet, but

- Bigger model (7 hidden layers, 650k units, 60M parameters)
- More data (10^6 images instead of 10^3)
- GPU implementation
- Better regularization and up-to-date tricks for training (Dropout)

A. Krizhevsky, I. Sutskever, and G. Hinton, ImageNet Classification with Deep Convolutional Neural Networks, NIPS 2012.

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Recap: VGGNet (2014/15)

- Main ideas
 - Deeper network
 - Stacked convolutional layers with smaller filters (+ nonlinearity)
 - Detailed evaluation of all components
- Results
 - Improved ILSVRC top-5 error rate to 6.7%.

| ConvNet Configurations | | | | |
|------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| A | B | C | D | E |
| 11 weight layers | A-LRN | 13 weight layers | 16 weight layers | 19 weight layers |
| conv-3-64 | conv-3-64 LRN | conv-3-64 conv-3-64 | conv-3-64 conv-3-64 | conv-3-64 conv-3-64 |
| conv-3-128 | conv-3-128 conv-3-128 | conv-3-128 conv-3-128 | conv-3-128 conv-3-128 | conv-3-128 conv-3-128 |
| conv-3-256 conv-3-256 | conv-3-256 conv-3-256 | conv-3-256 conv-3-256 | conv-3-256 conv-3-256 | conv-3-256 conv-3-256 |
| conv-3-512 conv-3-512 | conv-3-512 conv-3-512 | conv-3-512 conv-3-512 | conv-3-512 conv-3-512 | conv-3-512 conv-3-512 |
| conv-3-512 conv-3-512 | conv-3-512 conv-3-512 | conv-3-512 conv-3-512 | conv-3-512 conv-3-512 | conv-3-512 conv-3-512 |
| | | | | Mainly used |

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Recap: GoogLeNet (2014)

Inception module + copies

Auxiliary classification outputs for training the lower layers (deprecated)

Convolution Pooling Softmax Other

(b) Inception module with dimension reductions

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Recap: Residual Networks

AlexNet, 8 layers (ILSVRC 2012)

VGG, 19 layers (ILSVRC 2014)

ResNet, 152 layers (ILSVRC 2015)

- Core component
 - Skip connections bypassing each layer
 - Better propagation of gradients to the deeper layers
 - This makes it possible to train (much) deeper networks.

$$F(x) = \text{weight layer} \rightarrow \text{relu}$$

$$H(x) = F(x) + x \rightarrow \text{relu}$$

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Recap: Transfer Learning with CNNs

1. Train on ImageNet
2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier
3. If you have a medium sized dataset, "finetune" instead: use the old weights as initialization, train the full network or only some of the higher layers.

I.e., replace the Softmax layer at the end

Retrain bigger part of the network

Image

- conv-64
- conv-64
- maxpool
- conv-128
- conv-128
- maxpool
- conv-256
- conv-256
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- conv-512
- conv-512
- maxpool
- FC-4096
- FC-4096
- FC-1000
- softmax

Slide credit: Andrej Karpathy

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Repetition

- Image Processing Basics
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 - CNNs for Matching & RNNs
- 3D Reconstruction

MLPs

$$w_{kj}^{(\tau+1)} = w_{kj}^{(\tau)} - \eta \frac{\partial E(w)}{\partial w_{kj}}$$

Gradient Descent

ReLU

$$\text{Var}(W_i) = \frac{1}{n_{in}}$$

$$\text{Var}(W) = \frac{2}{n_{in}}$$

Dropout

Learning Rate

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Recap: Multi-Layer Perceptrons

- Deep network = Also learning the feature transformation

Output layer
Hidden layer
Mapping (learned!)
Input layer

- Output

$$y_k(\mathbf{x}) = g^{(2)} \left(\sum_{i=0}^h W_{ki}^{(2)} g^{(1)} \left(\sum_{j=0}^d W_{ij}^{(1)} x_j \right) \right)$$

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Recap: Backpropagation Algorithm

- General formulation (used in deep learning packages)
 - Convert the network into a computational graph.
 - Perform reverse-mode-differentiation this graph
 - Each new layer/module just needs to specify how it affects the
 - forward pass $\mathbf{y} = \text{module.fprop}(\mathbf{x})$
 - backward pass $\frac{\partial E}{\partial \mathbf{x}} = \text{module.bprop}(\frac{\partial E}{\partial \mathbf{y}})$

⇒ Very general framework, *any differentiable layer* can be used.

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Recap: Supervised Learning

- Two main steps
 - Computing the gradients for each weight (backprop)
 - Adjusting the weights in the direction of the gradient
- Gradient Descent: Basic update equation

$$w_{kj}^{(\tau+1)} = w_{kj}^{(\tau)} - \eta \left. \frac{\partial E(\mathbf{w})}{\partial w_{kj}} \right|_{\mathbf{w}^{(\tau)}}$$
- Important considerations
 - On what data do we want to apply this? ⇒ Minibatches
 - How should we choose the step size η (the learning rate)?
 - More advanced optimizers (Momentum, RMSProp, Adam, ...)

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Recap: Practical Considerations

- Vanishing gradients problem
 - In multilayer nets, gradients need to be propagated through many layers
 - The magnitudes of the gradients are often very different for the different layers, especially if the initial weights are small.
- ⇒ Gradients can get very small in the early layers of deep nets.
- When designing deep networks, we need to make sure gradients can be propagated throughout the network
 - By restricting the network depth (shallow networks are easier)
 - By very careful implementation (*numerics matter!*)
 - By choosing suitable nonlinearities (e.g., ReLU)
 - By performing proper initialization (Glorot, He)

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Recap: Glorot Initialization

[Glorot & Bengio, '10]

- Variance of neuron activations
 - Suppose we have an input X with n components and a linear neuron with random weights W that spits out a number Y .
 - We want the variance of the input and output of a unit to be the same, therefore $n \text{Var}(W_i)$ should be 1. This means

$$\text{Var}(W_i) = \frac{1}{n} = \frac{1}{n_{\text{in}}}$$
 - Or for the backpropagated gradient

$$\text{Var}(W_i) = \frac{1}{n_{\text{out}}}$$
 - As a compromise, Glorot & Bengio propose to use

$$\text{Var}(W) = \frac{2}{n_{\text{in}} + n_{\text{out}}}$$
- ⇒ Randomly sample the initial weights with this variance.

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Recap: He Initialization

[He et al., '15]

- Extension of Glorot Initialization to ReLU units
 - Use Rectified Linear Units (ReLU)

$$g(a) = \max\{0, a\}$$
 - Effect: gradient is propagated with a constant factor

$$\frac{\partial g(a)}{\partial a} = \begin{cases} 1, & a > 0 \\ 0, & \text{else} \end{cases}$$
- Same basic idea: Output should have the input variance
 - However, the Glorot derivation was based on tanh units, linearity assumption around zero does not hold for ReLU.
 - He et al. made the derivations, proposed to use instead

$$\text{Var}(W) = \frac{2}{n_{\text{in}}}$$

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[Ioffe & Szegedy '14]

Recap: Batch Normalization

- Motivation
 - Optimization works best if all inputs of a layer are normalized.
- Idea
 - Introduce intermediate layer that centers the activations of the previous layer per minibatch.
 - I.e., perform transformations on all activations and undo those transformations when backpropagating gradients
 - **Complication:** centering + normalization also needs to be done at test time, but minibatches are no longer available at that point.
 - Learn the normalization parameters to compensate for the expected bias of the previous layer (usually a simple moving average)
- Effect
 - Much improved convergence (but parameter values are important!)
 - Widely used in practice

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[Srivastava, Hinton '12]

Recap: Dropout

- Idea
 - Randomly switch off units during training.
 - Change network architecture for each data point, effectively training many different variants of the network.
 - When applying the trained network, multiply activations with the probability that the unit was set to zero.

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Recap: Reducing the Learning Rate

- Final improvement step after convergence is reached
 - Reduce learning rate by a factor of 10.
 - Continue training for a few epochs.
 - Do this 1-3 times, then stop training.
- Effect
 - Turning down the learning rate will reduce the random fluctuations in the error due to different gradients on different minibatches.
- *Be careful: Do not turn down the learning rate too soon!*
 - Further progress will be much slower after that.

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Recap: Data Augmentation

- Effect
 - Much larger training set
 - Robustness against expected variations
- During testing
 - When cropping was used during training, need to again apply crops to get same image size.
 - Beneficial to also apply flipping during test.
 - Applying several ColorPCA variations can bring another ~1% improvement, but at a significantly increased runtime.

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Image source: Lucas Bayer

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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
 - Convolutional Neural Networks (CNNs)
 - Deep Learning Background
 - CNNs for Object Detection
 - CNNs for Semantic Segmentation
 - CNNs for Matching & RNNs
- 3D Reconstruction

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Recap: R-CNN for Object Detection

R-CNN: Regions with CNN features

1. Input image
2. Extract region proposals (~2k)
3. Compute CNN features
4. Classify regions

- Key ideas
 - Extract region proposals (Selective Search)
 - Use a pre-trained/fine-tuned classification network as feature extractor (initially AlexNet, later VGGNet) on those regions

R. Girshick, J. Donahue, T. Darrell, and J. Malik, [Rich Feature Hierarchies for Accurate Object Detection and Semantic Segmentation](#), CVPR 2014 108

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Recap: R-CNN for Object Detection

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Slide credit: Ross Girshick

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Recap: Faster R-CNN

- One network, four losses
 - Remove dependence on external region proposal algorithm.
 - Instead, infer region proposals from same CNN.
 - Feature sharing
 - Joint training
 - ⇒ Object detection in a single pass becomes possible.

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Slide credit: Ross Girshick

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Recap: Mask R-CNN

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Slide credit: Feifei Li

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Recap: YOLO / SSD

- Idea: Directly go from image to detection scores
- Within each grid cell
 - Start from a set of anchor boxes
 - Regress from each of the B anchor boxes to a final box
 - Predict scores for each of C classes (including background)

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Slide credit: Feifei Li

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Repetition

- Image Processing Basics
- Segmentation & Grouping
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- Local Features & Matching
- Deep Learning
 - Convolutional Neural Networks (CNNs)
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- 3D Reconstruction

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Slide credit: B. Leibe

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Recap: Fully Convolutional Networks

- CNN
- FCN
 - convolutionalization
 - tabby cat heatmap
- Intuition
 - Think of FCNs as performing a sliding-window classification, producing a heatmap of output scores for each class

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Image source: Long, Shelhamer, Darrell

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Recap: Fully-Convolutional Networks

- Design a network as a sequence of convolutional layers
 - To make predictions for all pixels at once
 - Fully Convolutional Networks (FCNs)
 - All operations formulated as convolutions
 - Fully-connected layers become 1×1 convolutions
 - Advantage: can process arbitrarily sized images

Slide adapted from FeiFei Li
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Recap: Encoder-Decoder Architecture

- Design a network as a sequence of convolutional layers
 - With **downsampling** and **upsampling** inside the network!
 - **Downsampling**
 - Pooling, strided convolution
 - **Upsampling**
 - Unpooling or strided transpose convolution

Slide credit: FeiFei Li
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Recap: Skip Connections

- Encoder-Decoder Architecture with skip connections
 - Problem: downsampling loses high-resolution information
 - Use skip connections to preserve this higher-resolution information

Image source: Newell et al.
117

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Recap: FCNs for Human Pose Estimation

- Input data
 - Image
 - Keypoints
 - Labels
- Formulate pose estimation as a segmentation problem
 - Annotate images with keypoints for skeleton joints
 - Define a target disk around each keypoint with radius r
 - Set the ground-truth label to 1 within each such disk
 - Infer heatmaps for the joints as in semantic segmentation

Slide adapted from Georgia Gkioxari
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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
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 - Convolutional Neural Networks (CNNs)
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Siamese Networks
Triplet Loss
Recurrent Neural Networks

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Recap: Types of Models used for Matching Tasks

- Identification models (I)
 - Training: Multi-class classification loss
- Embedding models (E)
 - Training: Large-margin loss, Triplet loss
- Verification models (V)
 - Training: Two-class classification loss

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Triplet Loss Networks

- Learning a discriminative embedding
 - Present the network with triplets of examples
 - Negative
 - Anchor
 - Positive
 - Apply triplet loss to learn an embedding $f(\cdot)$ that groups the positive example closer to the anchor than the negative one.

$$\|f(x_i^a) - f(x_i^p)\|_2^2 < \|f(x_i^a) - f(x_i^n)\|_2^2$$

⇒ Used with great success in Google's FaceNet face identification

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Offline Hard Triplet Mining

- Considerable effort needed
 - Embed data with f_θ
 - Mine hard triplets
 - Update embedding f_θ
- Using the triplets for learning
 - Minibatch learning

This is a very wasteful design!

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Better: Online Hard Triplet Mining

- Core idea
 - The minibatch contains many more potential triplets than the ones that were mined!
 - Why not make use of those also?
- Possible improvement
 - Each member of another triplet becomes an additional negative candidate
 - But: need both hard negatives *and* hard positives!
- Better design
 - Sample K images from P classes (=people) for each minibatch
 - Triplets are only constructed within the minibatch

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Recap: Recurrent Neural Networks

- Up to now
 - Simple neural network structure: 1-to-1 mapping of inputs to outputs
- Recurrent Neural Networks
 - Generalize this to arbitrary mappings

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Recap: RNNs

- RNNs are regular NNs whose hidden units have additional forward connections over time.
 - You can **unroll** them to create a network that extends over time.
 - When you do this, keep in mind that the weights for the hidden units are shared between temporal layers.

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Recap: RNNs

- RNNs are very powerful, because they combine two properties:
 - Distributed hidden state that allows them to store a lot of information about the past efficiently.
 - Non-linear dynamics that allows them to update their hidden state in complicated ways.
- With enough neurons and time, RNNs can compute anything that can be computed by your computer.
- Training is more challenging (unrolled networks are deep)
 - See *Machine Learning* lecture for details...

Slide credit: Geoff Hinton

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Recap: Applications – Image Tagging

- Simple combination of CNN and RNN
 - Use CNN to define initial state h_0 of an RNN.
 - Use RNN to produce text description of the image.

Slide adapted from Andrej Karpathy. B. Leibe. 127

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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Deep Learning
- 3D Reconstruction
 - Epipolar Geometry and Stereo Basics
 - Camera Calibration & Uncalibrated Reconstruction
 - Structure-from-Motion

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Recap: What Is Stereo Vision?

- Generic problem formulation: given several images of the same object or scene, compute a representation of its 3D shape

Slide credit: Svetlana Lazebnik, Steve Seitz. B. Leibe. 129

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Recap: Depth with Stereo – Basic Idea

- Basic Principle: Triangulation
 - Gives reconstruction as intersection of two rays
 - Requires
 - Camera pose (calibration)
 - Point correspondence

Slide credit: Steve Seitz. B. Leibe. 130

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Recap: Epipolar Geometry

- Geometry of two views allows us to constrain where the corresponding pixel for some image point in the first view must occur in the second view.

- Epipolar constraint:
 - Correspondence for point p in Π must lie on the epipolar line l' in Π' (and vice versa).
 - Reduces correspondence problem to 1D search along conjugate epipolar lines.

Slide adapted from Steve Seitz. B. Leibe. 131

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Recap: Stereo Geometry With Calibrated Cameras

- Camera-centered coordinate systems are related by known rotation R and translation T :

$$\mathbf{X}' = R\mathbf{X} + \mathbf{T}$$

Slide credit: Kristen Grauman. B. Leibe. 132

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Recap: Essential Matrix

$\mathbf{X}' \cdot (\mathbf{T} \times \mathbf{R}\mathbf{X}) = 0$

$\mathbf{X}' \cdot (\mathbf{T}_x \mathbf{R}\mathbf{X}) = 0$

Let $\mathbf{E} = \mathbf{T}_x \mathbf{R}$

$\mathbf{X}'^T \mathbf{E} \mathbf{X} = 0$

• This holds for the rays p and p' that are parallel to the camera-centered position vectors \mathbf{X} and \mathbf{X}' , so we have: $\mathbf{p}'^T \mathbf{E} \mathbf{p} = 0$

• \mathbf{E} is called the essential matrix, which relates corresponding image points [Longuet-Higgins 1981]

Slide credit: Kristen Grauman B. Leibe 133

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Recap: Essential Matrix and Epipolar Lines

$\mathbf{p}'^T \mathbf{E} \mathbf{p} = 0$ Epipolar constraint: if we observe point p in one image, then its position p' in second image must satisfy this equation.

$\mathbf{l}' = \mathbf{E} \mathbf{p}$ is the coordinate vector representing the epipolar line for point p (i.e., the line is given by: $\mathbf{l}'^T \mathbf{x} = 0$)

$\mathbf{l} = \mathbf{E}^T \mathbf{p}'$ is the coordinate vector representing the epipolar line for point p'

Slide credit: Kristen Grauman B. Leibe 134

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Recap: Stereo Image Rectification

• In practice, it is convenient if image scanlines are the epipolar lines.

• Algorithm

- Reproject image planes onto a common plane parallel to the line between optical centers
- Pixel motion is horizontal after this transformation
- Two homographies (3x3 transforms), one for each input image reprojection

Slide adapted from Li Zhang C. Loop & Z. Zhang, Computing Rectifying Homographies for Stereo Vision, CVPR09 135

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Recap: Dense Correspondence Search

• For each pixel in the first image

- Find corresponding epipolar line in the right image
- Examine all pixels on the epipolar line and pick the best match (e.g. SSD, correlation)
- Triangulate the matches to get depth information

• This is easiest when epipolar lines are scanlines
⇒ Rectify images first

Adapted from Svetlana Lazebnik, Li Zhang 136

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Recap: Effect of Window Size

Want window large enough to have sufficient intensity variation, yet small enough to contain only pixels with about the same disparity.

Slide credit: Kristen Grauman B. Leibe Figures from Li Zhang 137

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Repetition

$\mathbf{K} = \begin{bmatrix} \alpha_x & s & x_0 \\ \alpha_y & y_0 \\ & & 1 \end{bmatrix}$ Camera models

Camera calibration

Triangulation

Essential matrix, Fundamental matrix

$\mathbf{x}^T \mathbf{E} \mathbf{x}' = 0$

$\mathbf{x}^T \mathbf{F} \mathbf{x}' = 0$

Eight-point algorithm

Structure-from-Motion

Slide credit: Kristen Grauman B. Leibe SVD! 138

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Recap: A General Point

- Equations of the form

$$Ax = 0$$
- How do we solve them? (always!)
 - Apply SVD

$$A = UDV^T = U \begin{bmatrix} d_{11} & & \\ & \ddots & \\ & & d_{NN} \end{bmatrix} \begin{bmatrix} v_{11} & \dots & v_{1N} \\ \vdots & \ddots & \vdots \\ v_{N1} & \dots & v_{NN} \end{bmatrix}^T$$

Singular values Singular vectors
- Singular values of A = square roots of the eigenvalues of A^TA.
- The solution of Ax=0 is the *nullspace* vector of A.
- This corresponds to the *smallest singular vector* of A.

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Recap: Camera Parameters

- Intrinsic parameters**
 - Principal point coordinates
 - Focal length
 - Pixel magnification factors
 - Skew (non-rectangular pixels)
 - Radial distortion
$$K = \begin{bmatrix} m_x & & & \\ & m_y & & \\ & & f & s \\ & & & 1 \end{bmatrix} \begin{bmatrix} \alpha_x & s' & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$
- Extrinsic parameters**
 - Rotation R
 - Translation t (both relative to world coordinate system)
- Camera projection matrix

$$P = K[R | t]$$
 - General pinhole camera: 9 DoF
 - CCD Camera with square pixels: 10 DoF
 - General camera: 11 DoF

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Recap: Calibrating a Camera

Goal

- Compute intrinsic and extrinsic parameters using observed camera data.

Main idea

- Place "calibration object" with known geometry in the scene
- Get correspondences
- Solve for mapping from scene to image: estimate $P = P_{int} P_{ext}$

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Recap: Camera Calibration (DLT Algorithm)

$$\begin{bmatrix} 0^T & X_1^T & -y_1 X_1^T \\ X_1^T & 0^T & -x_1 X_1^T \\ \dots & \dots & \dots \\ 0^T & X_n^T & -y_n X_n^T \\ X_n^T & 0^T & -x_n X_n^T \end{bmatrix} \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = 0 \quad Ap = 0$$

- P has 11 degrees of freedom.
- Two linearly independent equations per independent 2D/3D correspondence.
- Solve with SVD (similar to homography estimation)
 - Solution corresponds to smallest singular vector.
- 5 1/2 correspondences needed for a minimal solution.

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Recap: Triangulation – Lin. Alg. Approach

see Exercise 6.3!

$$\lambda_1 x_1 = P_1 X \quad x_1 \times P_1 X = 0 \quad [x_{1x}] P_1 X = 0$$

$$\lambda_2 x_2 = P_2 X \quad x_2 \times P_2 X = 0 \quad [x_{2x}] P_2 X = 0$$

- Two independent equations each in terms of three unknown entries of X.
- Stack equations and solve with SVD.
- This approach nicely generalizes to multiple cameras.

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Recap: Epipolar Geometry – Calibrated Case

Camera matrix: $[I | 0]$
 $X = (u, v, w, 1)^T$
 $x = (u, v, w)^T$

Camera matrix: $[R^T | -R^T t]$
 Vector x' in second coord. system has coordinates Rx' in the first one.

The vectors x , t , and Rx' are coplanar

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Recap: Epipolar Geometry – Calibrated Case

$x \cdot [t \times (Rx')] = 0 \Rightarrow x^T E x' = 0$ with $E = [t_x]_R$

Essential Matrix
(Longuet-Higgins, 1981)

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Slide credit: Svetlana Lazebnik
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Recap: Epipolar Geometry – Calibrated Case

$x \cdot [t \times (Rx')] = 0 \Rightarrow x^T E x' = 0$ with $E = [t_x]_R$

- $E x'$ is the epipolar line associated with x' ($l = E x'$)
- $E^T x$ is the epipolar line associated with x ($l' = E^T x$)
- $E e' = 0$ and $E^T e = 0$
- E is singular (rank two)
- E has five degrees of freedom (up to scale)

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Recap: Epipolar Geometry – Uncalibrated Case

- The calibration matrices K and K' of the two cameras are unknown
- We can write the epipolar constraint in terms of *unknown* normalized coordinates:

$\hat{x}^T E \hat{x}' = 0 \quad x = K \hat{x}, \quad x' = K' \hat{x}'$

Fundamental Matrix
(Faugeras and Luong, 1992)

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Recap: Epipolar Geometry – Uncalibrated Case

$\hat{x}^T E \hat{x}' = 0 \Rightarrow x^T F x' = 0$ with $F = K^{-T} E K'^{-1}$

$x = K \hat{x}$
 $x' = K' \hat{x}'$

Fundamental Matrix
(Faugeras and Luong, 1992)

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148

Recap: Epipolar Geometry – Uncalibrated Case

$\hat{x}^T E \hat{x}' = 0 \Rightarrow x^T F x' = 0$ with $F = K^{-T} E K'^{-1}$

- $F x'$ is the epipolar line associated with x' ($l = F x'$)
- $F^T x$ is the epipolar line associated with x ($l' = F^T x$)
- $F e' = 0$ and $F^T e = 0$
- F is singular (rank two)
- F has seven degrees of freedom

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Recap: The Eight-Point Algorithm

$x = (u, v, 1)^T, \quad x' = (u', v', 1)^T$

$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow (uu', uv', u, vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$

$\begin{bmatrix} uu'_1 & uv'_1 & u_1 & vv'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3 u'_3 & u_3 v'_3 & u_3 & v_3 v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4 u'_4 & u_4 v'_4 & u_4 & v_4 v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5 u'_5 & u_5 v'_5 & u_5 & v_5 v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6 u'_6 & u_6 v'_6 & u_6 & v_6 v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7 u'_7 & u_7 v'_7 & u_7 & v_7 v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8 u'_8 & u_8 v'_8 & u_8 & v_8 v'_8 & v_8 & u'_8 & v'_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$

$\begin{matrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{matrix} = 0$

$\sum_{i=1}^N (x_i^T F x'_i)^2$

- 1.) Solve with SVD. This minimizes $\sum_{i=1}^N (x_i^T F x'_i)^2$
- 2.) Enforce rank-2 constraint using SVD

- Problem: poor numerical conditioning

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see Exercise 6.1!

Recap: Normalized Eight-Point Alg.

- Center the image data at the origin, and scale it so the mean squared distance between the origin and the data points is 2 pixels.
- Use the eight-point algorithm to compute F from the normalized points.
- Enforce the rank-2 constraint using SVD.

Set d_{33} to zero and reconstruct F

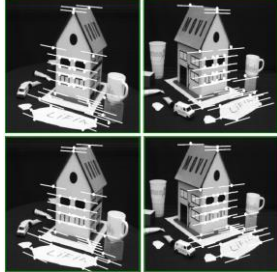
$$F = UDV^T = U \begin{bmatrix} d_{11} & & & \\ & d_{22} & & \\ & & d_{33} & \\ & & & \dots \end{bmatrix} \begin{bmatrix} v_{11} & \dots & v_{13} \\ \vdots & \ddots & \vdots \\ v_{31} & \dots & v_{33} \\ \vdots & \ddots & \vdots \end{bmatrix}^T$$
- Transform fundamental matrix back to original units: if T and T' are the normalizing transformations in the two images, then the fundamental matrix in original coordinates is $T^T F T'$.

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Slide credit: Svetlana Lazebnik, B. Leibe, Hartley, 1995

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Recap: Comparison of Estimation Algorithms



| | 8-point | Normalized 8-point | Nonlinear least squares |
|-------------|-------------|--------------------|-------------------------|
| Av. Dist. 1 | 2.33 pixels | 0.92 pixel | 0.86 pixel |
| Av. Dist. 2 | 2.18 pixels | 0.85 pixel | 0.80 pixel |

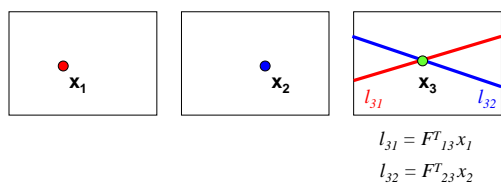
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Slide credit: Svetlana Lazebnik

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Recap: Epipolar Transfer

- Assume the epipolar geometry is known
- Given projections of the same point in two images, how can we compute the projection of that point in a third image?



$$l_{31} = F_{13}^T x_1$$

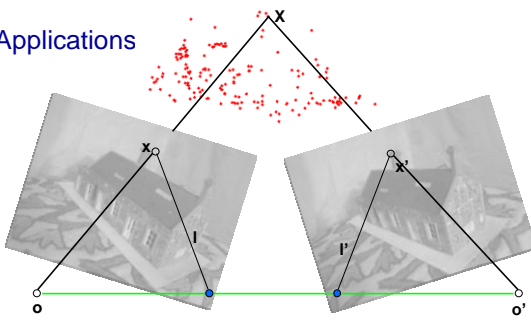
$$l_{32} = F_{23}^T x_2$$

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Applications



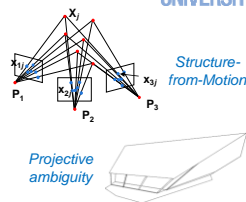
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Repetition

- Image Processing Basics
- Segmentation & Grouping
- Object Recognition
- Local Features & Matching
- Object Categorization
- 3D Reconstruction
 - Epipolar Geometry and Stereo Basics
 - Camera Calibration & Uncalibrated Reconstruction
 - Structure-from-Motion

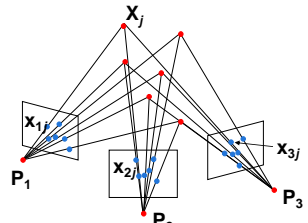


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Recap: Structure from Motion



- Given: m images of n fixed 3D points

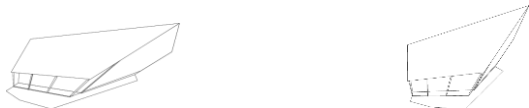
$$x_{ij} = P_i X_j, \quad i = 1, \dots, m, \quad j = 1, \dots, n$$
- Problem: estimate m projection matrices P_i and n 3D points X_j from the mn correspondences x_{ij}

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Slide credit: Svetlana Lazebnik, B. Leibe





Recap: Structure from Motion Ambiguity

- If we scale the entire scene by some factor k and, at the same time, scale the camera matrices by the factor of $1/k$, the projections of the scene points in the image remain exactly the same.
- More generally: if we transform the scene using a transformation Q and apply the inverse transformation to the camera matrices, then the images do not change

$$\mathbf{x} = \mathbf{P}\mathbf{X} = (\mathbf{P}\mathbf{Q}^{-1})\mathbf{Q}\mathbf{X}$$



Recap: Hierarchy of 3D Transformations

| | | | |
|---------------------|--|---|--------------------------------------|
| Projective 15dof | $\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{v}^T & v \end{bmatrix}$ |  | Preserves intersection and tangency |
| Affine 12dof | $\begin{bmatrix} \mathbf{A} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$ |  | Preserves parallelism, volume ratios |
| Similarity 7dof | $\begin{bmatrix} s\mathbf{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$ |  | Preserves angles, ratios of length |
| Euclidean 6dof | $\begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0}^T & 1 \end{bmatrix}$ |  | Preserves angles, lengths |

- With no constraints on the camera calibration matrix or on the scene, we get a *projective* reconstruction.
- Need additional information to *upgrade* the reconstruction to affine, similarity, or Euclidean.

Any More Questions?

Good luck for the exam!