RWTH AACHEN UNIVERSITY

Computer Vision II - Lecture 10

Particle Filters (The Gritty Details)

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Bastian Leibe
RWTH Aachen
http://www.vision.rwth-aachen.de

leibe@vision.rwth-aachen.de

Announcement

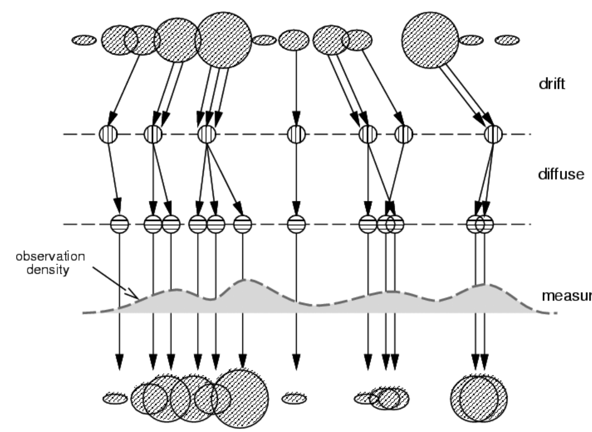
- Problems with exam registration fixed...
  - ...for Master CS and Master SSE
  - You should now be able to register
  - I extended the registration deadline until this Friday (30.05.)

- Exchange students can register directly with us
  - If registration is not possible via ZPA

- Please let us know if problems persist.

Course Outline

- Single-Object Tracking
  - Background modeling
  - Template based tracking
  - Color based tracking
  - Contour based tracking
  - Tracking by online classification
  - Tracking-by-detection
- Bayesian Filtering
  - Kalman filters
  - Particle filters
  - Case studies
- Multi-Object Tracking
- Articulated Tracking



Today: Beyond Gaussian Error Models

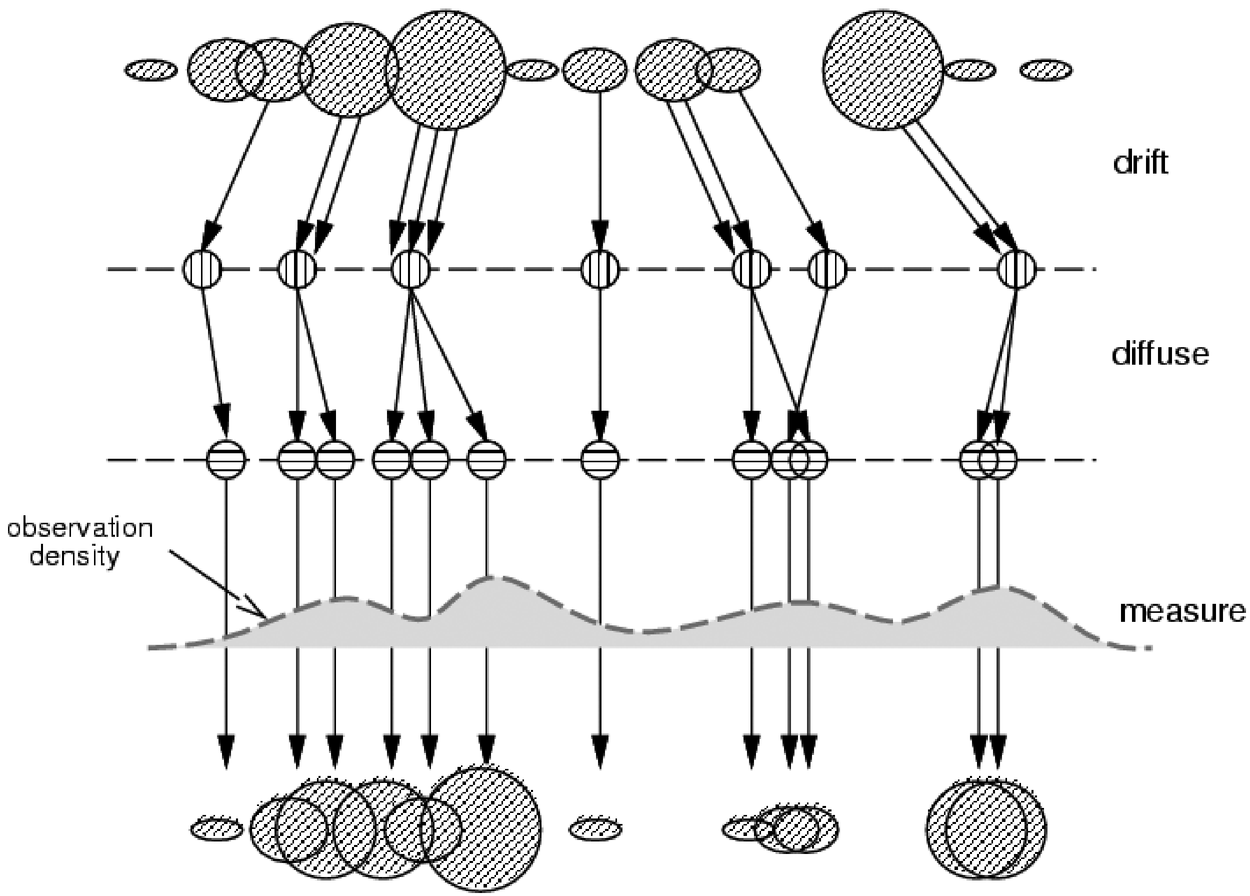


Figure from Isard & Blake

Topics of This Lecture

- Recap: Extended Kalman Filter
  - Detailed algorithm
- Particle Filters: Detailed Derivation
  - Recap: Basic idea
  - Importance Sampling
  - Sequential Importance Sampling (SIS)
  - Transitional prior
  - Resampling
  - Generic Particle Filter
  - Sampling Importance Resampling (SIR)

Recap: Kalman Filter

- Algorithm summary
  - Assumption: linear model

$$\begin{aligned}\mathbf{x}_t &= \mathbf{D}_t \mathbf{x}_{t-1} + \varepsilon_t \\ \mathbf{y}_t &= \mathbf{M}_t \mathbf{x}_t + \delta_t\end{aligned}$$

  - Prediction step

$$\begin{aligned}\mathbf{x}_t^- &= \mathbf{D}_t \mathbf{x}_{t-1}^+ \\ \mathbf{\Sigma}_t^- &= \mathbf{D}_t \mathbf{\Sigma}_{t-1}^+ \mathbf{D}_t^T + \mathbf{\Sigma}_{d_t}\end{aligned}$$

  - Correction step

$$\begin{aligned}\mathbf{K}_t &= \mathbf{\Sigma}_t^- \mathbf{M}_t^T \left(\mathbf{M}_t \mathbf{\Sigma}_t^- \mathbf{M}_t^T + \mathbf{\Sigma}_{m_t} \right)^{-1} \\ \mathbf{x}_t^+ &= \mathbf{x}_t^- + \mathbf{K}_t \left(\mathbf{y}_t - \mathbf{M}_t \mathbf{x}_t^- \right) \\ \mathbf{\Sigma}_t^+ &= \left(\mathbf{I} - \mathbf{K}_t \mathbf{M}_t \right) \mathbf{\Sigma}_t^-\end{aligned}$$

Recap: Extended Kalman Filter (EKF)

- Algorithm summary
  - Nonlinear model

$$\mathbf{x}_t = \mathbf{g}\left(\mathbf{x}_{t-1}\right) + \varepsilon_t$$
$$\mathbf{y}_t = \mathbf{h}\left(\mathbf{x}_t\right) + \delta_t$$

  - Prediction step

$$\mathbf{x}_t^- = \mathbf{g}\left(\mathbf{x}_{t-1}^+\right)$$
$$\mathbf{\Sigma}_t^- = \mathbf{G}_t \mathbf{\Sigma}_{t-1}^+ \mathbf{G}_t^T + \mathbf{\Sigma}_{d_t}$$

  - Correction step

$$\mathbf{K}_t = \mathbf{\Sigma}_t^- \mathbf{H}_t^T \left(\mathbf{H}_t \mathbf{\Sigma}_t^- \mathbf{H}_t^T + \mathbf{\Sigma}_{m_t}\right)^{-1}$$
$$\mathbf{x}_t^+ = \mathbf{x}_t^- + \mathbf{K}_t \left(\mathbf{y}_t - \mathbf{h}\left(\mathbf{x}_t^-\right)\right)$$
$$\mathbf{\Sigma}_t^+ = \left(\mathbf{I} - \mathbf{K}_t \mathbf{H}_t\right) \mathbf{\Sigma}_t^-$$

with the Jacobians

$$\mathbf{G}_t = \left.\frac{\partial \mathbf{g}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_{t-1}^+}$$
$$\mathbf{H}_t = \left.\frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}_t^-}$$

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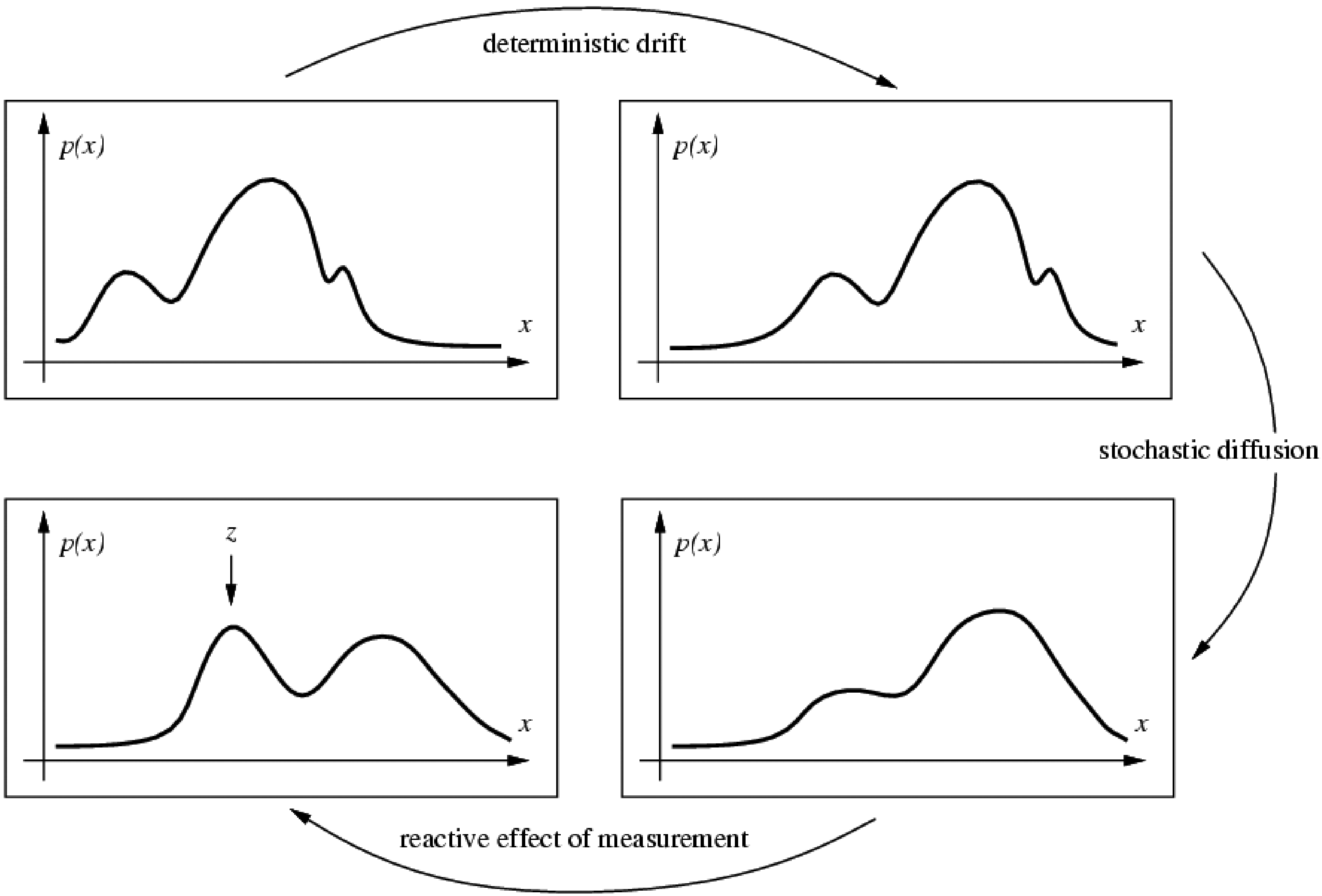
- Recap: Extended Kalman Filter
  - Detailed algorithm
- Particle Filters: Detailed Derivation
  - Recap: Basic idea
  - Importance Sampling
  - Sequential Importance Sampling (SIS)
  - Transitional prior
  - Resampling
  - Generic Particle Filter
  - Sampling Importance Resampling (SIR)

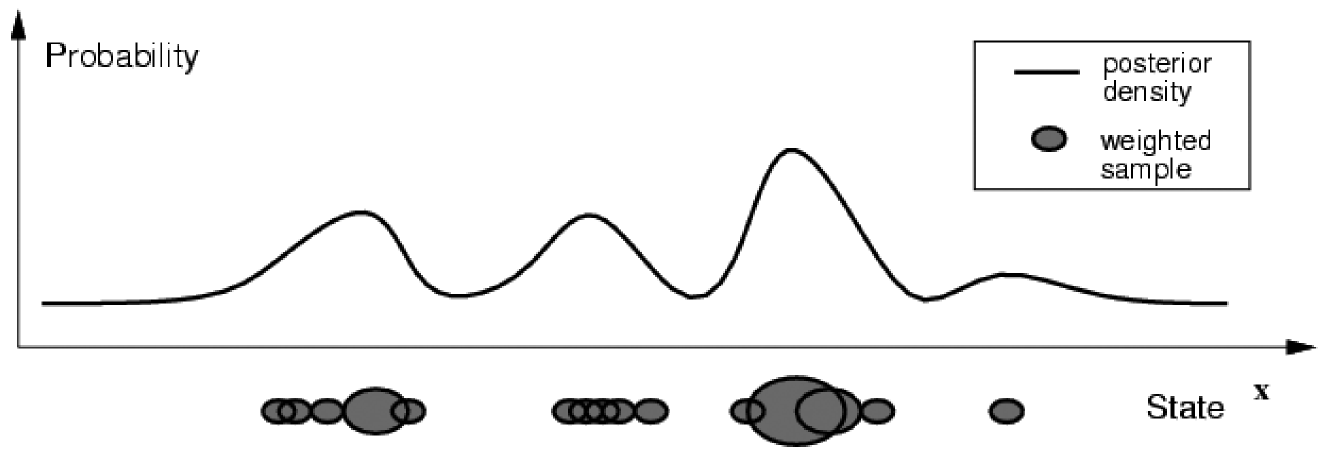
Recap: Propagation of General Densities



Slide credit: Svetlana Lazebnik

Figure from Isard & Blake

Recap: Factored Sampling

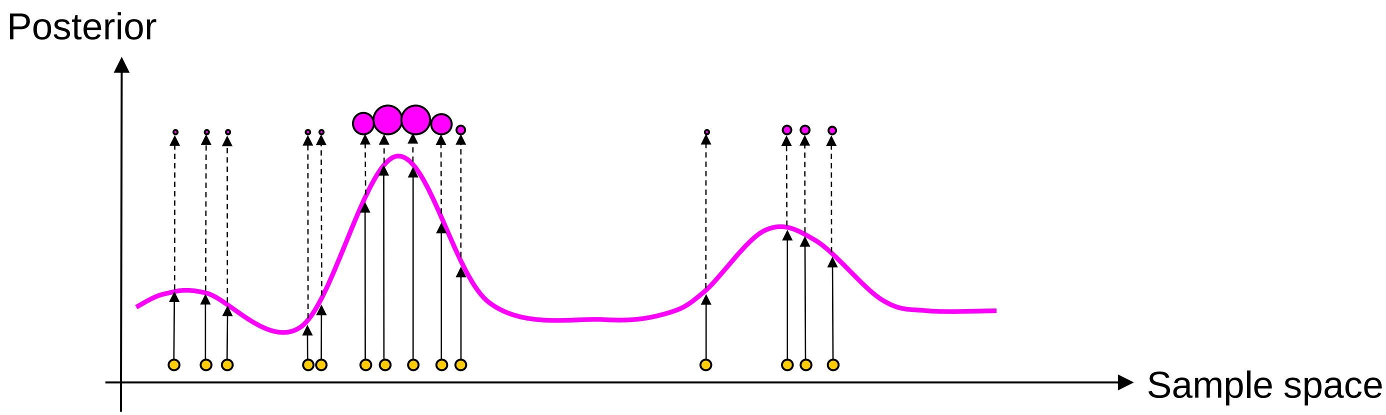


- **Idea: Represent state distribution non-parametrically**
  - **Prediction: Sample points from prior density for the state,** $P(X)$
  - **Correction: Weight the samples according to** $P(Y|X)$

$$P(X_t \mid y_0, \ldots, y_t) = \frac{P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1})}{\int P(y_t \mid X_t) P(X_t \mid y_0, \ldots, y_{t-1}) dX_t}$$

Particle Filtering

- **Many variations, one general concept:**
  - ***Represent the posterior pdf by a set of randomly chosen weighted samples (particles)***



  - **Randomly Chosen = Monte Carlo (MC)**
  - **As the number of samples become very large - the characterization becomes an equivalent representation of the true pdf.**

Particle filtering

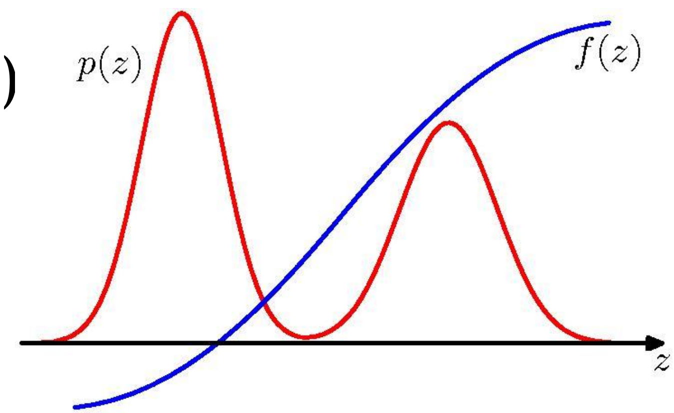
- Compared to Kalman Filters and their extensions
  - Can represent any arbitrary distribution
  - Multimodal support
  - Keep track of as many hypotheses as there are particles
  - Approximate representation of complex model rather than exact representation of simplified model

- The basic building-block: *Importance Sampling*

Recap: Monte-Carlo Sampling

- **Objective:**
  - Evaluate expectation of a function $f(\mathbf{z})$ w.r.t. a probability distribution $p(\mathbf{z})$.

$$\mathbb{E}[f] = \int f(\mathbf{z})p(\mathbf{z})d\mathbf{z}$$



- **Monte Carlo Sampling idea**
  - Draw L independent samples $\mathbf{z}^{(l)}$ with l = 1,…,L from $p(\mathbf{z})$.
  - This allows the expectation to be approximated by a finite sum

$$\hat{f} = \frac{1}{L}\sum_{l=1}^{L} f(\mathbf{z}^l)$$

  - As long as the samples $\mathbf{z}^{(l)}$ are drawn independently from $p(\mathbf{z})$, then

$$\mathbb{E}[\hat{f}] = \mathbb{E}[f]$$

$\Rightarrow$ Unbiased estimate, independent of the dimension of $\mathbf{z}$!

Monte Carlo Integration

- We can use the same idea for computing integrals
  - Assume we are trying to estimate a complicated integral of a function f over some domain D:

$$F = \int_D f(\vec{x})d\vec{x}$$

  - Also assume there exists some PDF p defined over D. Then

$$F = \int_D f(\vec{x})d\vec{x} = \int_D \frac{f(\vec{x})}{p(\vec{x})} p(\vec{x})d\vec{x}$$

  - For any pdf p over D, the following holds

$$\int_D \frac{f(\vec{x})}{p(\vec{x})} p(\vec{x})d\vec{x} = E\left[\frac{f(\vec{x})}{p(\vec{x})}\right], x \sim p$$

Monte Carlo Integration

- **Idea (cont'd)**
  - Now, if we have i.i.d random samples $x_1, \ldots, x_N$ sampled from p, then we can approximate the expectation

$$E\left[\frac{f(\vec{x})}{p(\vec{x})}\right]$$

  - by

$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(\vec{x}_i)}{p(\vec{x}_i)}$$

  - Guaranteed by law of large numbers:

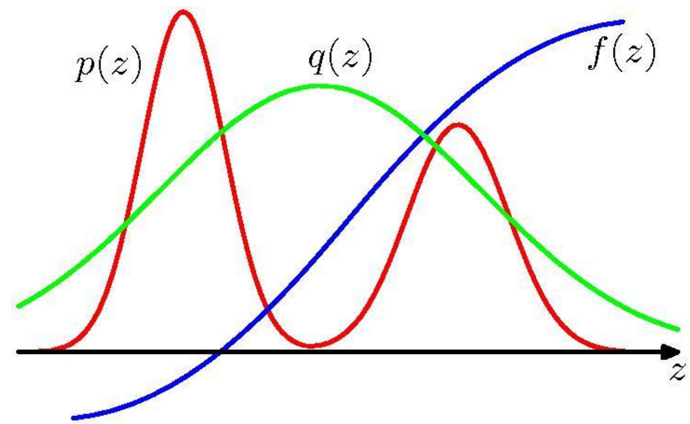
$$N \rightarrow \infty, F_N \xrightarrow{a.s} E\left[\frac{f(\vec{x})}{p(\vec{x})}\right] = F$$

  - Since it guides sampling, p is often called a **proposal distribution**.

Importance Sampling

- **Let's consider an example**

$$F_N = \frac{1}{N}\sum_{i=1}^{N}\frac{f(\vec{x}_i)}{p(\vec{x}_i)}$$



  - f/p is the **importance weight** of a sample.
  - *What can go wrong here?*

- **What if $p(x)=0$?**
  - If p is very small, then f/p can get arbitrarily large!

  $\Rightarrow$ Design p such that f/p is bounded.
  - Effect: get more samples in "important" areas of f, i.e., where f is large.

Image source: C.M. Bishop, 2006

Proposal Distributions: Other Uses

- **Similar Problem**
  - For many distributions, sampling directly from $p(\mathbf{z})$ is difficult.
  - But we can often easily *evaluate* $p(\mathbf{z})$ (up to some normalization factor Z_p):

$$p(\mathbf{z}) = \frac{1}{Z_p}\tilde{p}(\mathbf{z})$$

- **Idea**
  - Take some simpler distribution $q(\mathbf{z})$ as proposal distribution from which we can draw samples and which is non-zero.

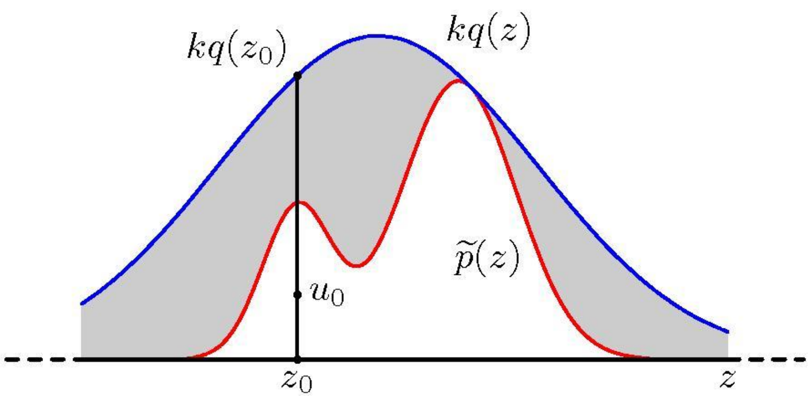


Image source: C.M. Bishop, 2006

Recap: Importance Sampling

- **Idea**
  - Use a proposal distribution $q(\mathbf{z})$ from which it is easy to draw samples and which is close in shape to f.
  - Express expectations in the form of a finite sum over samples $\{\mathbf{z}^{(l)}\}$ drawn from $q(\mathbf{z})$.

$$\mathbb{E}[f] = \int f(\mathbf{z})p(\mathbf{z})d\mathbf{z} = \int f(\mathbf{z})\frac{p(\mathbf{z})}{q(\mathbf{z})}q(\mathbf{z})d\mathbf{z}$$

$$\simeq \frac{1}{L}\sum_{l=1}^{L}\frac{p(\mathbf{z}^{(l)})}{q(\mathbf{z}^{(l)})}f(\mathbf{z}^{(l)})$$

  - with **importance weights**

$$r_l = \frac{p(\mathbf{z}^{(l)})}{q(\mathbf{z}^{(l)})}$$

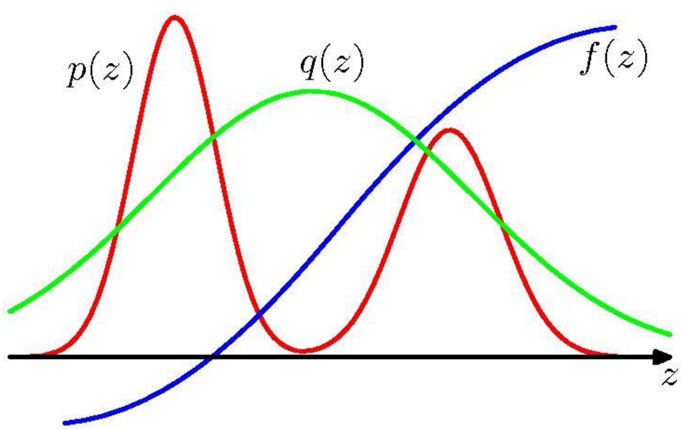
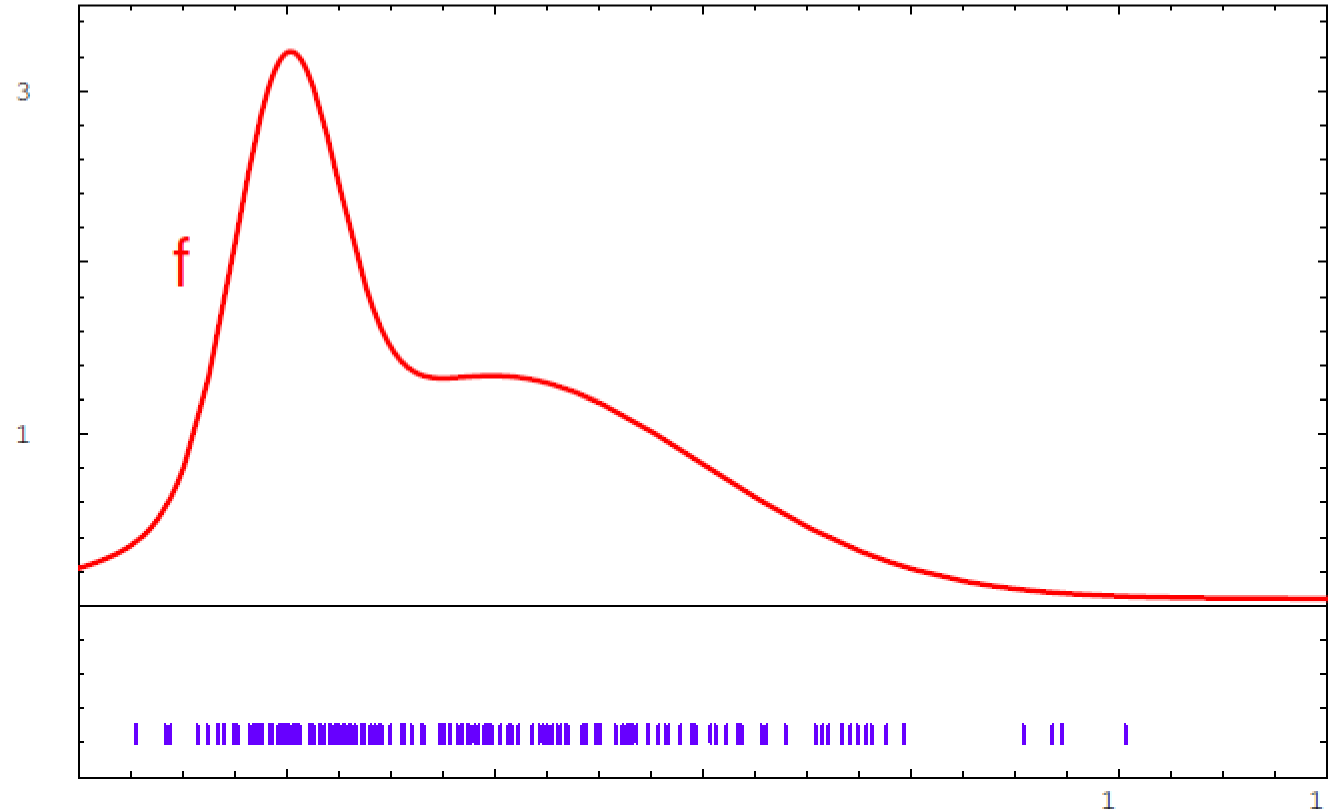


Image source: C.M. Bishop, 2006

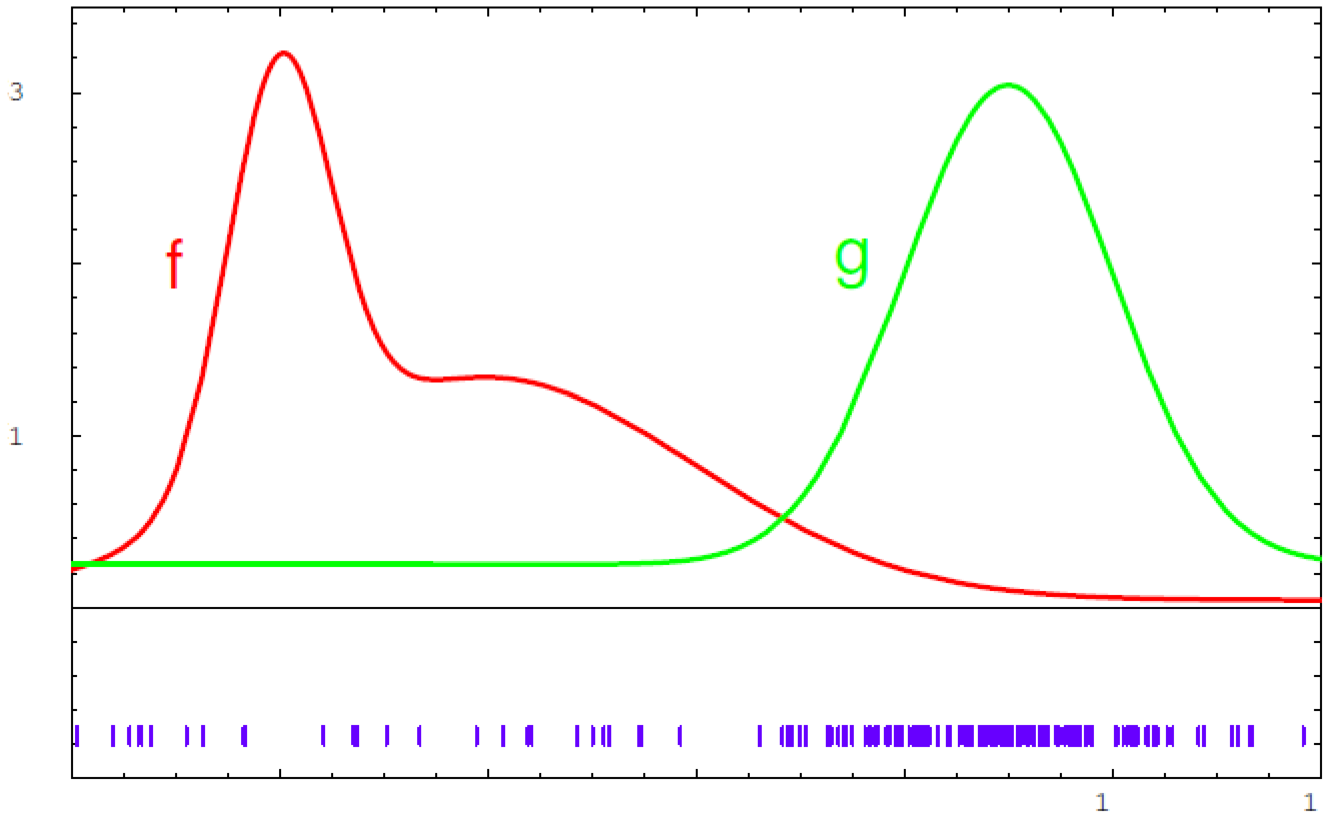
Illustration of Importance Factors



- **Goal: Approximate target density f**

Figure source: Thrun, Burgard, Fox

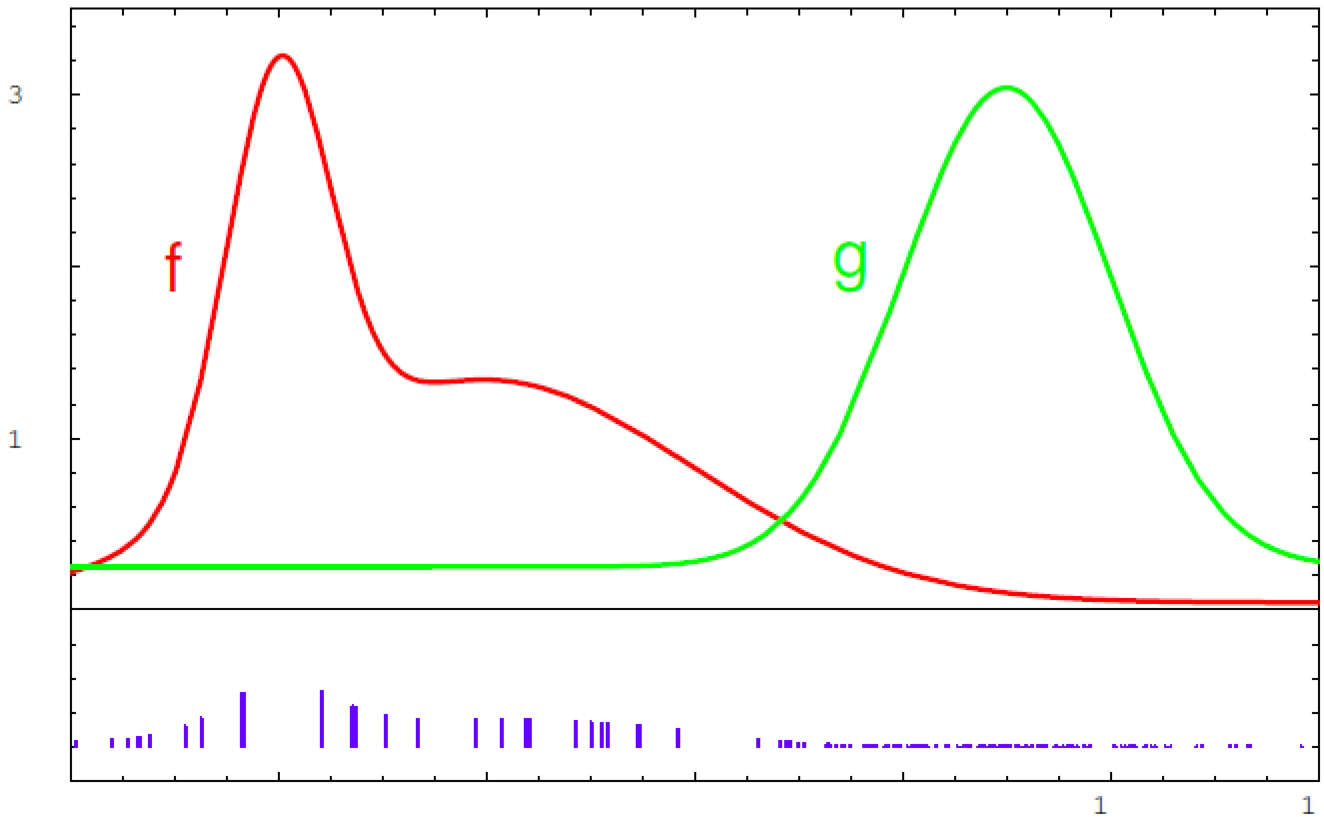
Illustration of Importance Factors



- **Goal: Approximate target density f**
  - **Instead of sampling from f directly, we can only sample from g.**

Figure source: Thrun, Burgard, Fox

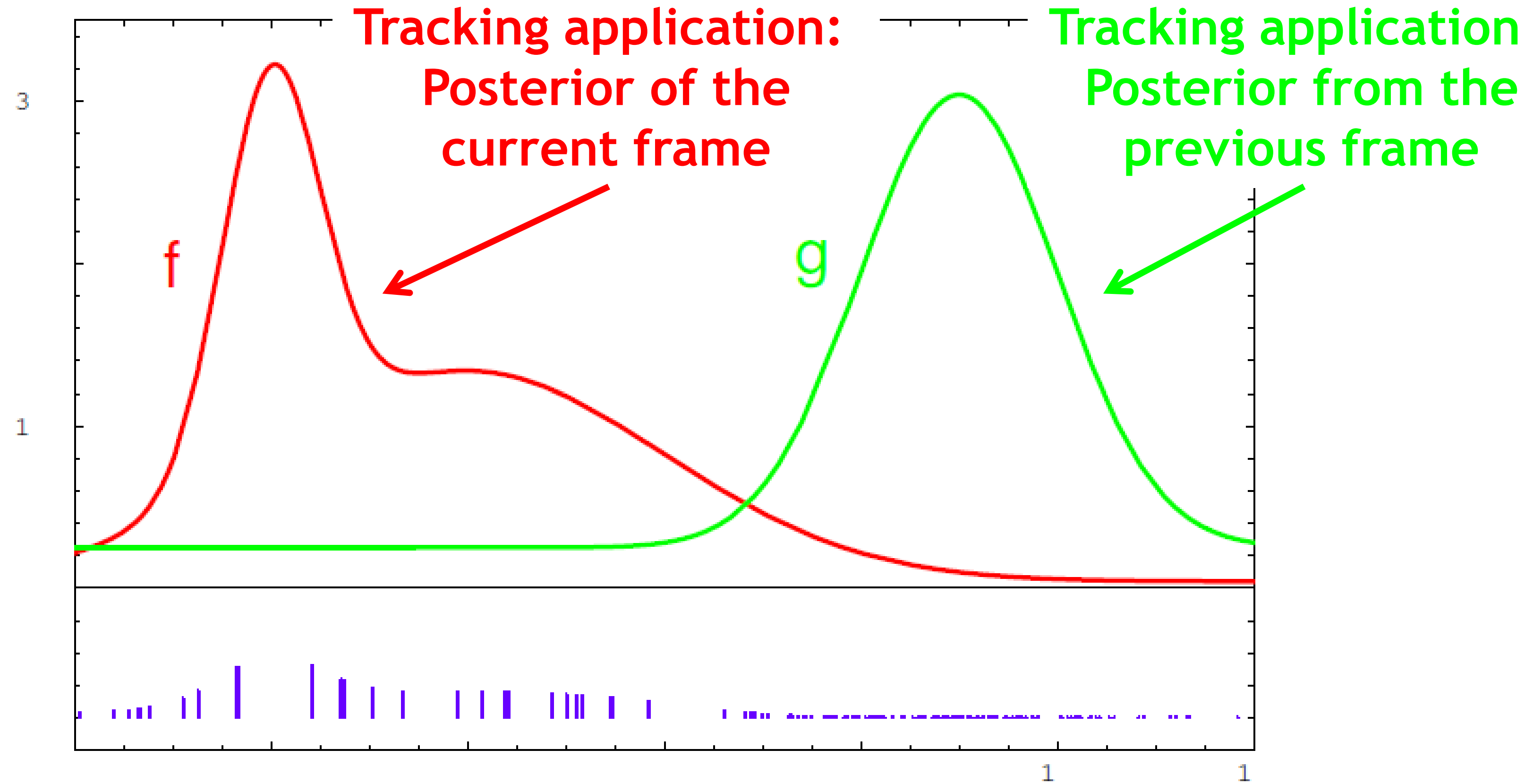
Illustration of Importance Factors



- **Goal: Approximate target density f**
  - **Instead of sampling from f directly, we can only sample from g.**
  - **A sample of f is obtained by attaching the weight f/g to each sample $\mathbf{x}$.**

Figure source: Thrun, Burgard, Fox

Interpretation for Tracking



- **Goal: Approximate target density f**
  - **Instead of sampling from f directly, we can only sample from g.**
  - **A sample of f is obtained by attaching the weight f/g to each sample $\mathbf{x}$.**

Figure source: Thrun, Burgard, Fox

Importance Sampling for Bayesian Estimation

$$\begin{aligned}\mathbb{E}[f(X)] &= \int_X f(\mathbf{x}_{0:t})p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})d\mathbf{x}_{0:t} \\ &= \int_X f(\mathbf{x}_{0:t})\frac{p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})}{q(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})}q(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})d\mathbf{x}_{0:t}\end{aligned}$$

- **Applying Importance Sampling**
  - **Characterize the posterior pdf using a set of samples (particles) and their weights**

$$\left\{\mathbf{x}_{0:t}^i, w_t^i\right\}_{i=1}^N$$

  - **Then the joint posterior is approximated by**

$$p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) \approx \sum_{i=1}^N w_t^i \delta(\mathbf{x}_{0:t} - \mathbf{x}_{0:t}^i)$$

Importance Sampling for Bayesian Estimation

$$\mathbb{E}[f(X)] = \int_X f(\mathbf{x}_{0:t}) p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) d\mathbf{x}_{0:t}$$

$$= \int_X f(\mathbf{x}_{0:t}) \frac{p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})}{q(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})} q(\mathbf{x}_{0:t}|\mathbf{y}_{1:t}) d\mathbf{x}_{0:t}$$

- **Applying Importance Sampling**
  - **Draw the samples from the importance density $q(\mathbf{x}_{0:t} \mid \mathbf{y}_{1:t})$ with importance weights**

    $$w_t^i \propto \frac{p(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})}{q(\mathbf{x}_{0:t}|\mathbf{y}_{1:t})}$$

  - **Sequential update (after some calculation)**
    - **Particle update** $\mathbf{x}_t \sim q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)$
    - **Weight update** $w_t^i = w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i) p(\mathbf{x}_t^i|\mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)}$

Sequential Importance Sampling Algorithm

$$\text{function}\ \left[\{\mathbf{x}_t^i, w_t^i\}_{i=1}^N\right] = SIS\left[\{\mathbf{x}_{t-1}^i, w_{t-1}^i\}_{i=1}^N, \mathbf{y}_t\right]$$

$\eta = 0$ — **Initialize**

for $i = 1{:}N$

$$\mathbf{x}_t^i \sim q(\mathbf{x}_t | \mathbf{x}_{t-1}^i, \mathbf{y}_t)$$

Sample from proposal pdf

$$w_t^i = w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i) p(\mathbf{x}_t^i|\mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)}$$

Update weights

$$\eta = \eta + w_t^i$$

Update norm. factor

end

for $i = 1{:}N$

$$w_t^i = w_t^i / \eta$$

Normalize weights

end

Slide adapted from Michael Rubinstein

Sequential Importance Sampling Algorithm

$$\textbf{function} \left[\{\mathbf{x}_t^i, w_t^i\}_{i=1}^N\right] = SIS\left[\{\mathbf{x}_{t-1}^i, w_{t-1}^i\}_{i=1}^N, \mathbf{y}_t\right]$$

$\eta = 0$ — **Initialize**

for $i = 1{:}N$

$\mathbf{x}_t^i \sim q(\mathbf{x}_t | \mathbf{x}_{t-1}^i, \mathbf{y}_t)$ — **Sample from proposal pdf**

$$w_t^i = w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i)p(\mathbf{x}_t^i|\mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)}$$ — **Update weights**

$\eta = \eta + w_t^i$ — **Update norm. factor**

end

for $i = 1{:}N$

$w_t^i = w_t^i / \eta$ — **Normalize weights**

end

For a concrete algorithm, we need to define the importance density $q(.|.)$!

Choice of Importance Density

- **Most common choice**
  - Transitional prior

$$q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)$$

  - With this choice, the weight update equation simplifies to

$$\begin{aligned} w_t^i &= w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i)p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)}{q(\mathbf{x}_t|\mathbf{x}_{t-1}^i, \mathbf{y}_t)} \\ &= w_{t-1}^i \frac{p(\mathbf{y}_t|\mathbf{x}_t^i)\cancel{p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)}}{\cancel{p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)}} \\ &= w_{t-1}^i p(\mathbf{y}_t|\mathbf{x}_t^i) \end{aligned}$$

SIS Algorithm with Transitional Prior

function $\left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = SIS\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^N, \mathbf{y}_t\right]$

$\eta = 0$ — **Initialize**

for $i = 1{:}N$

$\mathbf{x}_t^i \sim p(\mathbf{x}_t | \mathbf{x}_{t-1}^i)$ — **Sample from proposal pdf**

$w_t^i = w_{t-1}^i p(\mathbf{y}_t | \mathbf{x}_t^i)$ — **Update weights**

$\eta = \eta + w_t^i$ — **Update norm. factor**

end

for $i = 1{:}N$

$w_t^i = w_t^i / \eta$ — **Normalize weights**

end

SIS Algorithm with Transitional Prior

$$\textbf{function} \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = SIS\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^N, \mathbf{y}_t\right]$$

$\eta = 0$ — **Initialize**

$\textbf{for } i = 1{:}N$

  $\textit{Draw } \varepsilon_t^i \textit{ from noise distribution}$

  $\mathbf{x}_t^i = \mathbf{g}\left(\mathbf{x}_{t-1}^i\right) + \varepsilon_t^i$ — **Sample from proposal pdf**

  $w_t^i = w_{t-1}^i p(\mathbf{y}_t | \mathbf{x}_t^i)$ — **Update weights**

  $\eta = \eta + w_t^i$ — **Update norm. factor**

$\textbf{end}$

$\textbf{for } i = 1{:}N$

  $w_t^i = w_t^i / \eta$ — **Normalize weights**

$\textbf{end}$

Slide adapted from Michael Rubinstein

The Degeneracy Phenomenon

- Unavoidable problem with SIS
  - After a few iterations, most particles have negligible weights.
  - Large computational effort for updating particles with very small contribution to $p(\mathbf{x}_t \mid \mathbf{y}_{1:t})$.

- Measure of degeneracy
  - Effective sample size

$$N_{\mathit{eff}} = \frac{1}{\sum_{i=1}^{N} (w_t^i)^2}$$

  - Uniform: $N_{eff} = N$
  - Severe degeneracy: $N_{eff} = 1$

Resampling

- **Idea**
  - **Eliminate particles with low importance weights and increase the number of particles with high importance weight.**

$$\{\mathbf{x}_t^i, w_t^i\}_{i=1}^N \rightarrow \left\{\mathbf{x}_t^{i*}, \frac{1}{N}\right\}_{i=1}^N$$

  - **The new set is generated by sampling with replacement from the discrete representation of $p(\mathbf{x}_t \mid \mathbf{y}_{1:t})$ such that**

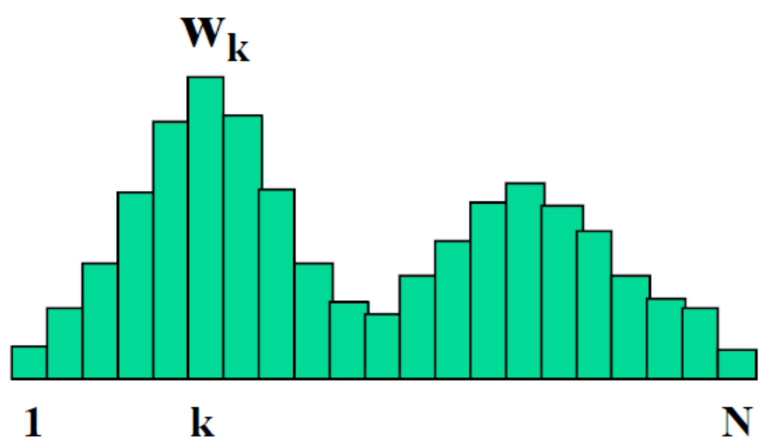
$$Pr\left\{\mathbf{x}_t^{i*} = \mathbf{x}_t^j\right\} = w_t^j$$

Resampling

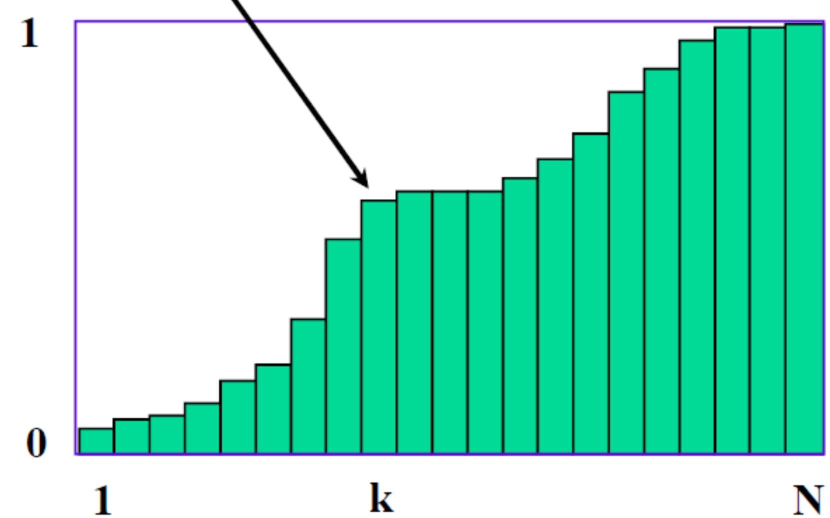
- How to do that in practice?
  - We want to resample $\left\{\mathbf{x}_t^i\right\}_{i=1}^N$ from the discrete pdf given by the weighted samples $\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N$.
  - I.e., we want to draw N new samples $\left\{\mathbf{x}_t^i\right\}_{i=1}^N$ with replacement where the probability of drawing $\mathbf{x}_t^j$ is given by w_t^j.
- There are many algorithms for this
  - We will look at two simple algorithms here...

Inverse Transform Sampling

- **Idea**
  - It is easy to sample from a discrete distribution using the cumulative distribution function $F(x) = p(X \leq x)$.



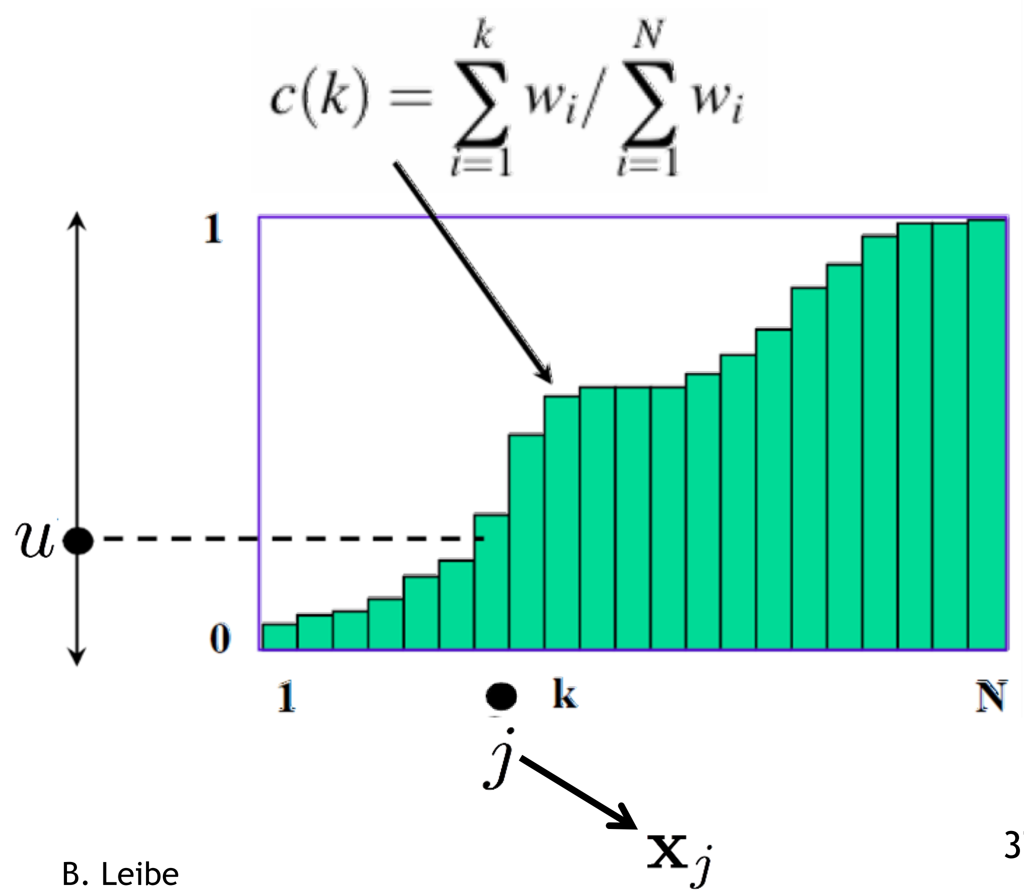
$$c(k) = \sum_{i=1}^{k} w_i / \sum_{i=1}^{N} w_i$$



Inverse Transform Sampling

- **Idea**
  - It is easy to sample from a discrete distribution using the cumulative distribution function $F(x) = p(X \leq x)$.

- **Procedure**
  1. Generate uniform u in the range $[0,1]$.
  2. Visualize a horizontal line intersecting the bars.
  3. If index of intersected bar is j, output new sample $\mathbf{x}_j$.



More Efficient Approach

- **From Arulampalam paper:**

Algorithm 2: Resampling Algorithm

$[\{\mathbf{x}_k^{j*}, w_k^j, i^j\}_{j=1}^{N_s}]$ = RESAMPLE $[\{\mathbf{x}_k^i, w_k^i\}_{i=1}^{N_s}]$

- Initialize the CDF: $c_1 = 0$
- FOR $i = 2: N_s$
  - Construct CDF: $c_i = c_{i-1} + w_k^i$
- END FOR
- Start at the bottom of the CDF: $i = 1$
- Draw a starting point: $u_1 \sim \mathbb{U}[0, N_s^{-1}]$
- FOR $j = 1: N_s$
  - Move along the CDF: $u_j = u_1 + N_s^{-1}(j-1)$
  - WHILE $u_j > c_i$
    * $i = i + 1$
  - END WHILE
  - Assign sample: $\mathbf{x}_k^{j*} = \mathbf{x}_k^i$
  - Assign weight: $w_k^j = N_s^{-1}$
  - Assign parent: $i^j = i$
- END FOR

Basic idea: choose one initial small random number; deterministically sample the rest by "crawling" up the cdf. This is $\mathcal{O}(N)$!

Generic Particle Filter

$$\textbf{function} \quad \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = PF\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^N, \mathbf{y}_t\right]$$

$$\textit{Apply SIS filtering} \quad \left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = SIS\left[\left\{\mathbf{x}_{t-1}^i, w_{t-1}^i\right\}_{i=1}^N, \mathbf{y}_t\right]$$

$$\textit{Calculate } N_{eff}$$

$$\textbf{if} \quad N_{eff} < N_{thr}$$

$$\left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right] = RESAMPLE\left[\left\{\mathbf{x}_t^i, w_t^i\right\}_{i=1}^N\right]$$

$$\textbf{end}$$

- **We can also apply resampling selectively**
  - **Only resample when it is needed, i.e., N_{eff} is too low.**

  $\Rightarrow$ Avoids drift when there the tracked state is stationary.

Other Variant of the Algorithm

function $[\mathcal{X}_t] = SIR\,[\mathcal{X}_{t-1}, \mathbf{y}_t]$

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$ — **Initialize**

for $i = 1{:}N$

$\quad Sample\ \ \mathbf{x}_t^i \sim p(\mathbf{x}_t|\mathbf{x}_{t-1}^i)$ — **Generate new samples**

$\quad w_t^i = p(\mathbf{y}_t|\mathbf{x}_t^i)$ — **Update weights**

end

for $i = 1{:}N$

$\quad Draw\ i\ with\ probability \propto w_t^i$

$\quad Add\ \mathbf{x}_t^i\ \ to\ \mathcal{X}_t$

Resample

end

Other Variant of the Algorithm

function $[\mathcal{X}_t] = SIR\,[\mathcal{X}_{t-1}, \mathbf{y}_t]$

$\bar{\mathcal{X}}_t = \mathcal{X}_t = \emptyset$

for $i = 1{:}N$

$\quad Sample\;\; \mathbf{x}_t^i \sim p(\mathbf{x}_t | \mathbf{x}_{t-1}^i)$

$\quad w_t^i = p(\mathbf{y}_t | \mathbf{x}_t^i)$

end

for $i = 1{:}N$

$\quad Draw\ i\ with\ probability \propto w_t^i$

$\quad Add\ \mathbf{x}_t^i\ \ to\ \mathcal{X}_t$

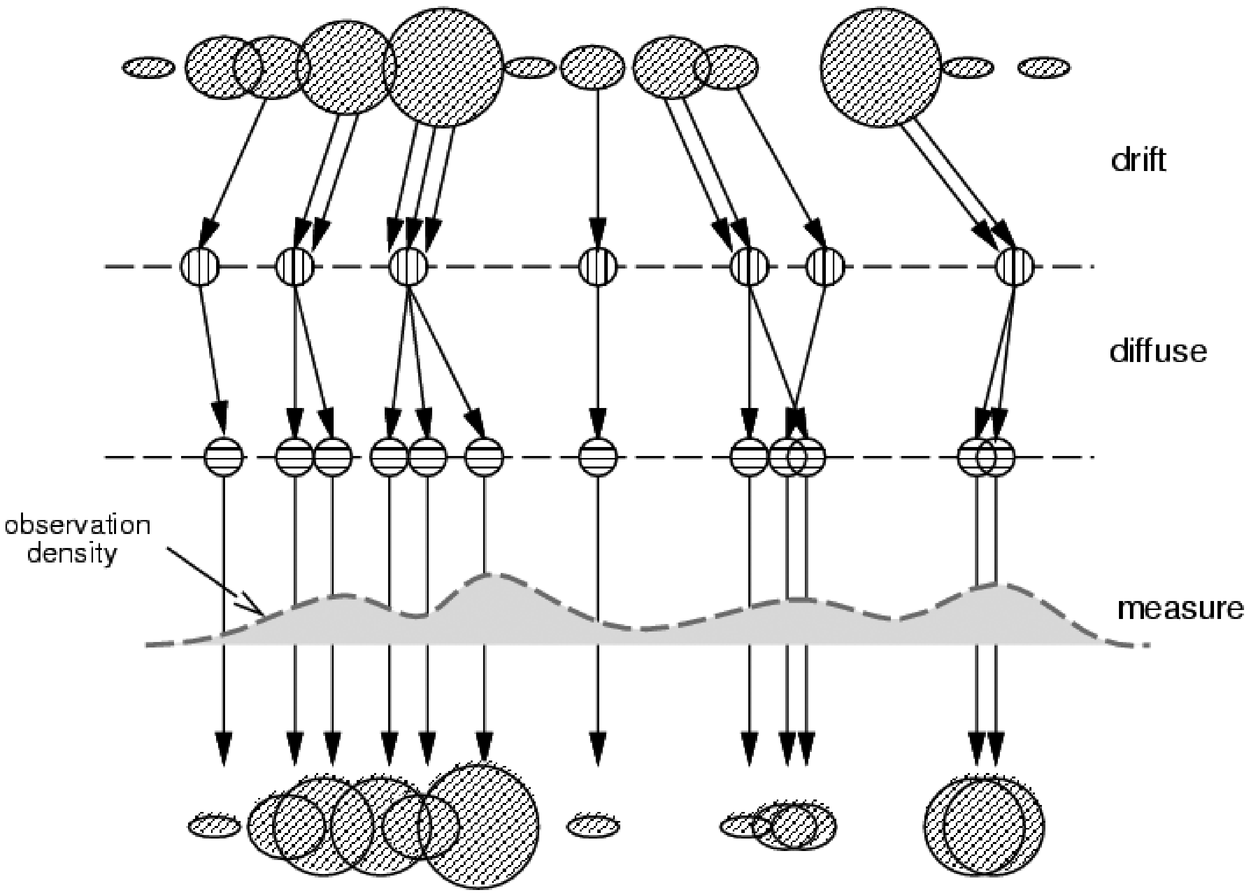
end

Important property:

Particles are distributed according to pdf from previous time step.

Particles are distributed according to posterior from this time step.

Particle Filtering: Condensation Algorithm



Start with weighted samples from previous time step

Sample and shift according to dynamics model

Spread due to randomness; this is predicted density $p(x_t | y_{t-1})$

Weight the samples according to observation density

Arrive at corrected density estimate $p(x_t | y_t)$

M. Isard and A. Blake, CONDENSATION -- conditional density propagation for visual tracking, IJCV 29(1):5-28, 1998

Figure source: M. Isard & A. Blake

Summary: Particle Filtering

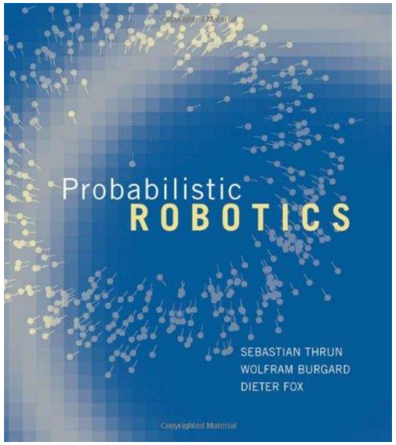
- **Pros:**
  - Able to represent arbitrary densities
  - Converging to true posterior even for non-Gaussian and nonlinear system
  - Efficient: particles tend to focus on regions with high probability
  - Works with many different state spaces
    - E.g. articulated tracking in complicated joint angle spaces
  - Many extensions available

Summary: Particle Filtering

- **<u>Cons / Caveats:</u>**
  - #Particles is important performance factor
    - Want as few particles as possible for efficiency.
    - But need to cover state space sufficiently well.
  - Worst-case complexity grows exponentially in the dimensions
  - Multimodal densities possible, but still single object
    - Interactions between multiple objects require special treatment.
    - Not handled well in the particle filtering framework (state space explosion).

References and Further Reading

- A good description of Particle Filters can be found in Ch.4.3 of the following book
  - S. Thrun, W. Burgard, D. Fox. Probabilistic Robotics. MIT Press, 2006.



- A good tutorial on Particle Filters
  - M.S. Arulampalam, S. Maskell, N. Gordon, T. Clapp. A Tutorial on Particle Filters for Online Nonlinear/Non-Gaussian Bayesian Tracking. In *IEEE Transactions on Signal Processing*, Vol. 50(2), pp. 174-188, 2002.
- The CONDENSATION paper
  - M. Isard and A. Blake, CONDENSATION - conditional density propagation for visual tracking, IJCV 29(1):5-28, 1998