Computer Vision II - Lecture 8

Tracking with Linear Dynamic Models

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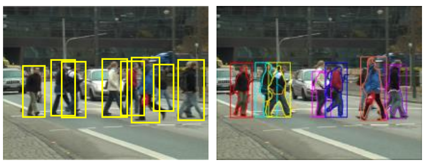
Course Outline

- Single-Object Tracking
  - Background modeling
  - Template based tracking
  - Color based tracking
  - Contour based tracking
  - Tracking by online classification
  - Tracking-by-detection
- Bayesian Filtering
  - Kalman filter
  - Particle filter
- Multi-Object Tracking
- Articulated Tracking

Image source: Helmut Grabner, Disney/Pixar

Recap: Tracking-by-Detection

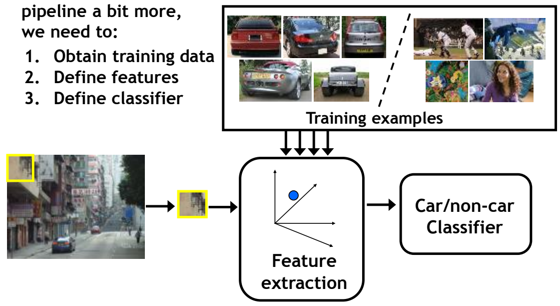


- Main ideas
  - Apply a generic object detector to find objects of a certain class
  - Based on the detections, extract object appearance models
  - Link detections into trajectories

Recap: Sliding-Window Object Detection

Fleshing out this pipeline a bit more, we need to:

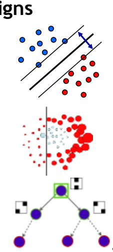
1. Obtain training data
2. Define features
3. Define classifier



Slide credit: Kristen Grauman

Recap: Object Detector Design

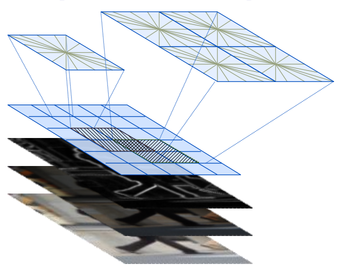
- In practice, the classifier often determines the design.
  - Types of features
  - Speedup strategies
- We'll look at 3 state-of-the-art detector designs
  - Based on SVMs
    → Last lecture
  - Based on Boosting
    → Last lecture
  - Based on Random Forests
    → Postponed to a later slot...

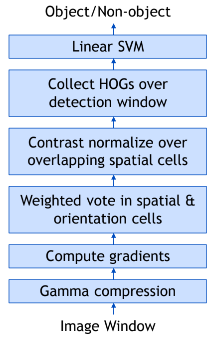


Recap: Histograms of Oriented Gradients (HOG)

- Holistic object representation
  - Localized gradient orientations

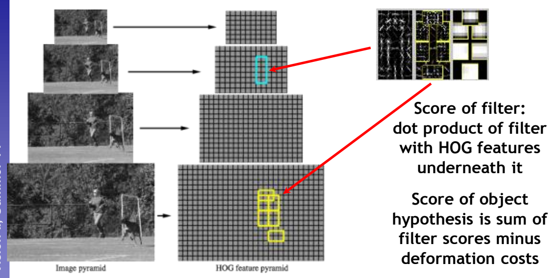
[..., ..., ..., ...]





Slide adapted from Navneet Dalal

Recap: Deformable Part-based Model (DPM)



Score of filter: dot product of filter with HOG features underneath it

Score of object hypothesis is sum of filter scores minus deformation costs

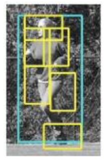
- Multiscale model captures features at two resolutions

Recap: DPM Hypothesis Score

$$\text{score}(p_0, \ldots, p_n) = \underbrace{\sum_{i=0}^{n} F_i \cdot \phi(H, p_i)}_{\text{"data term"}} - \underbrace{\sum_{i=1}^{n} d_i \cdot (dx_i^2, dy_i^2)}_{\text{"spatial prior"}}$$

"data term" — filters

"spatial prior" — displacements, deformation parameters

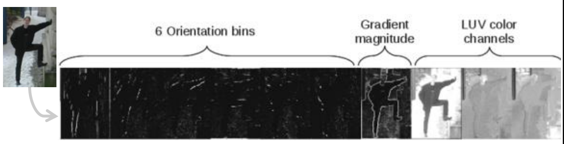


$$\text{score}(z) = \beta \cdot \Psi(H, z)$$

β: concatenation filters and deformation parameters

$\Psi(H, z)$: concatenation of HOG features and part displacement features

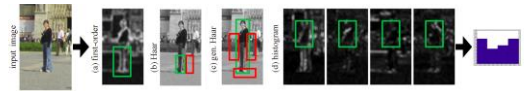
Recap: Integral Channel Features



- Generalization of Haar Wavelet idea from Viola-Jones
  - Instead of only considering intensities, also take into account other feature channels (gradient orientations, color, texture).
  - Still efficiently represented as integral images.

P. Dollar, Z. Tu, P. Perona, S. Belongie. Integral Channel Features, BMVC'09.

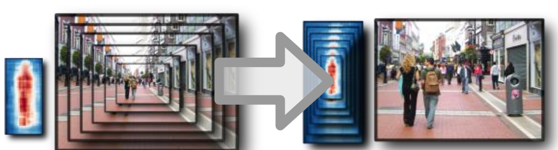
Recap: Integral Channel Features



- Generalize also block computation
  - 1st order features:
    - Sum of pixels in rectangular region.
  - 2nd-order features:
    - Haar-like difference of sum-over-blocks
  - Generalized Haar:
    - More complex combinations of weighted rectangles
  - Histograms
    - Computed by evaluating local sums on quantized images.

Recap: VeryFast Detector

- Idea 1: Invert the relation



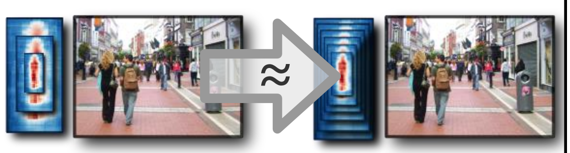
1 model, 50 image scales

50 models, 1 image scale

R. Benenson, M. Mathias, R. Timofte, L. Van Gool. Pedestrian Detection at 100 Frames per Second, CVPR'12.

Recap: VeryFast Detector

- Idea 2: Reduce training time by feature interpolation

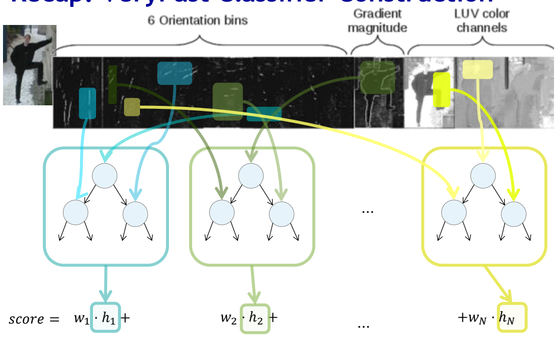


5 models, 1 image scale

50 models, 1 image scale

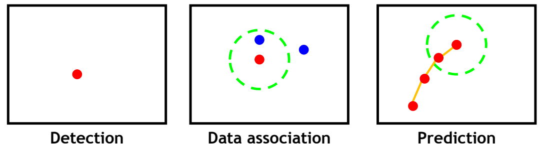
- Shown to be possible for Integral Channel features
  - P. Dollár, S. Belongie, Perona. The Fastest Pedestrian Detector in the West, BMVC 2010.

Recap: VeryFast Classifier Construction



- Ensemble of short trees, learned by AdaBoost

Elements of Tracking



- Detection — Last lecture
  - *Where are candidate objects?*
- Data association
  - *Which detection corresponds to which object?*
- Prediction — Today's topic
  - *Where will the tracked object be in the next time step?*

Today: Tracking with Linear Dynamic Models

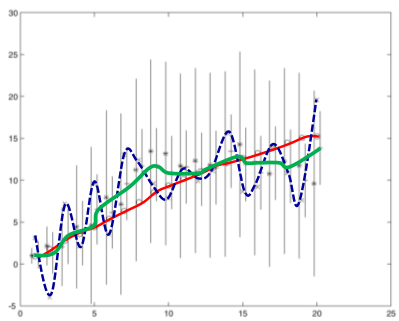


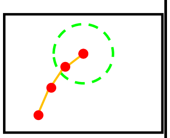
Figure from Forsyth & Ponce

Topics of This Lecture

- Tracking with Dynamics
  - Detection vs. Tracking
  - Tracking as probabilistic inference
  - Prediction and Correction
- Linear Dynamic Models
  - Zero velocity model
  - Constant velocity model
  - Constant acceleration model
- The Kalman Filter
  - Kalman filter for 1D state
  - General Kalman filter
  - Limitations

Tracking with Dynamics

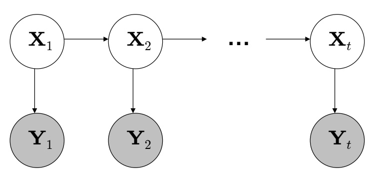
- Key idea
  - Given a model of expected motion, predict where objects will occur in next frame, even before seeing the image.



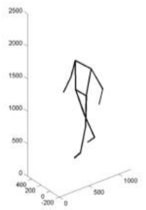
- Goals
  - Restrict search for the object
  - Improved estimates since measurement noise is reduced by trajectory smoothness.
- Assumption: continuous motion patterns
  - Camera is not moving instantly to new viewpoint.
  - Objects do not disappear and reappear in different places.
  - Gradual change in pose between camera and scene.

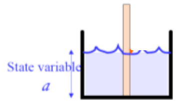
General Model for Tracking

- Representation
  - The moving object of interest is characterized by an underlying *state* $\mathbf{X}$.
  - State $\mathbf{X}$ gives rise to *measurements* or *observations* $\mathbf{Y}$.
  - At each time t, the state changes to $\mathbf{X}_t$ and we get a new observation $\mathbf{Y}_t$.



State vs. Observation





- Hidden state : parameters of interest
- Measurement: what we get to directly observe

Tracking as Inference

- Inference problem
  - The hidden state consists of the true parameters we care about, denoted $\mathbf{X}$.
  - The measurement is our noisy observation that results from the underlying state, denoted $\mathbf{Y}$.
  - At each time step, state changes (from $\mathbf{X}_{t-1}$ to $\mathbf{X}_t$) and we get a new observation $\mathbf{Y}_t$.
- Our goal: recover most likely state $\mathbf{X}_t$ given
  - All observations seen so far.
  - Knowledge about dynamics of state transitions.

Steps of Tracking

- Prediction: What is the next state of the object given past measurements?

$$P(X_t | Y_0 = y_0, \dots, Y_{t-1} = y_{t-1})$$

- Correction: Compute an updated estimate of the state from prediction and measurements.

$$P(X_t | Y_0 = y_0, \dots, Y_{t-1} = y_{t-1}, Y_t = y_t)$$

- Tracking can be seen as the process of propagating the posterior distribution of state given measurements across time.

Simplifying Assumptions

- Only the immediate past matters

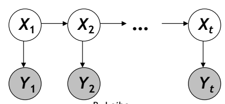
$$P(X_t | X_0, \dots, X_{t-1}) = \boxed{P(X_t | X_{t-1})}$$

Dynamics model

- Measurements depend only on the current state

$$P(Y_t | X_0, Y_0 \dots, X_{t-1}, Y_{t-1}, X_t) = \boxed{P(Y_t | X_t)}$$

Observation model



Tracking as Induction

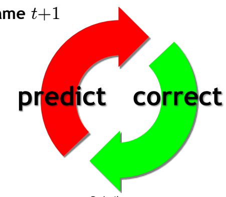
- Base case:
  - Assume we have initial prior that predicts state in absence of any evidence: $P(\mathbf{X}_0)$
  - At the first frame, *correct* this given the value of $\mathbf{Y}_0 = \mathbf{y}_0$

$$P(X_0 | Y_0 = y_0) = \frac{P(y_0 | X_0) P(X_0)}{P(y_0)} \propto P(y_0 | X_0) P(X_0)$$

Posterior prob. of state given measurement — Likelihood of measurement — Prior of the state

Tracking as Induction

- Base case:
  - Assume we have initial prior that predicts state in absence of any evidence: $P(\mathbf{X}_0)$
  - At the first frame, *correct* this given the value of $\mathbf{Y}_0 = \mathbf{y}_0$
- Given corrected estimate for frame t:
  - Predict for frame $t+1$
  - Correct for frame $t+1$



Induction Step: Prediction

- Prediction involves representing $P(X_t|y_0,\dots,y_{t-1})$ given $P(X_{t-1}|y_0,\dots,y_{t-1})$

$$P(X_t|y_0,\dots,y_{t-1})$$
$$=\int P(X_t,X_{t-1}|y_0,\dots,y_{t-1})dX_{t-1}$$

Law of total probability

$$P(A)=\int P(A,B)dB$$

Induction Step: Prediction

- Prediction involves representing $P(X_t|y_0,\dots,y_{t-1})$ given $P(X_{t-1}|y_0,\dots,y_{t-1})$

$$P(X_t|y_0,\dots,y_{t-1})$$
$$=\int P(X_t,X_{t-1}|y_0,\dots,y_{t-1})dX_{t-1}$$
$$=\int P(X_t\mid X_{t-1},y_0,\dots,y_{t-1})P(X_{t-1}\mid y_0,\dots,y_{t-1})dX_{t-1}$$

Conditioning on X_{t-1}

$$P(A,B)=P(A|B)P(B)$$

Induction Step: Prediction

- Prediction involves representing $P(X_t|y_0,\dots,y_{t-1})$ given $P(X_{t-1}|y_0,\dots,y_{t-1})$

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$$=\int P(X_t\mid X_{t-1})P(X_{t-1}\mid y_0,\dots,y_{t-1})dX_{t-1}$$

Independence assumption

Induction Step: Correction

- Correction involves computing $P(X_t|y_0,\dots,y_t)$ given predicted value $P(X_t|y_0,\dots,y_{t-1})$

$$P(X_t|y_0,\dots,y_t)$$
$$=\frac{P(y_t\mid X_t,y_0,\dots,y_{t-1})P(X_t\mid y_0,\dots,y_{t-1})}{P(y_t\mid y_0,\dots,y_{t-1})}$$

Bayes rule

$$P(A|B)=\frac{P(B|A)P(A)}{P(B)}$$

Induction Step: Correction

- Correction involves computing $P(X_t|y_0,\dots,y_t)$ given predicted value $P(X_t|y_0,\dots,y_{t-1})$

$$P(X_t|y_0,\dots,y_t)$$
$$=\frac{P(y_t\mid X_t,y_0,\dots,y_{t-1})P(X_t\mid y_0,\dots,y_{t-1})}{P(y_t\mid y_0,\dots,y_{t-1})}$$
$$=\frac{P(y_t\mid X_t)P(X_t\mid y_0,\dots,y_{t-1})}{P(y_t\mid y_0,\dots,y_{t-1})}$$

Independence assumption
(observation y_t depends only on state X_t)

Induction Step: Correction

- Correction involves computing $P(X_t|y_0,\dots,y_t)$ given predicted value $P(X_t|y_0,\dots,y_{t-1})$

$$P(X_t|y_0,\dots,y_t)$$
$$=\frac{P(y_t\mid X_t,y_0,\dots,y_{t-1})P(X_t\mid y_0,\dots,y_{t-1})}{P(y_t\mid y_0,\dots,y_{t-1})}$$
$$=\frac{P(y_t\mid X_t)P(X_t\mid y_0,\dots,y_{t-1})}{P(y_t\mid y_0,\dots,y_{t-1})}$$
$$=\frac{P(y_t\mid X_t)P(X_t\mid y_0,\dots,y_{t-1})}{\int P(y_t\mid X_t)P(X_t\mid y_0,\dots,y_{t-1})dX_t}$$

Conditioning on X_t

Summary: Prediction and Correction

- **Prediction:**

$$P(X_t \mid y_0,\ldots,y_{t-1}) = \int \underbrace{P(X_t \mid X_{t-1})}_{\text{Dynamics model}} \underbrace{P(X_{t-1} \mid y_0,\ldots,y_{t-1})}_{\text{Corrected estimate from previous step}} dX_{t-1}$$

Summary: Prediction and Correction

- **Prediction:**

$$P(X_t \mid y_0,\ldots,y_{t-1}) = \int \underbrace{P(X_t \mid X_{t-1})}_{\text{Dynamics model}} \underbrace{P(X_{t-1} \mid y_0,\ldots,y_{t-1})}_{\text{Corrected estimate from previous step}} dX_{t-1}$$

- **Correction:**

$$P(X_t \mid y_0,\ldots,y_t) = \frac{\overbrace{P(y_t \mid X_t)}^{\text{Observation model}} \overbrace{P(X_t \mid y_0,\ldots,y_{t-1})}^{\text{Predicted estimate}}}{\int P(y_t \mid X_t) P(X_t \mid y_0,\ldots,y_{t-1}) dX_t}$$

Topics of This Lecture

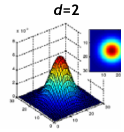
- Tracking with Dynamics
  - Detection vs. Tracking
  - Tracking as probabilistic inference
  - Prediction and Correction
- **Linear Dynamic Models**
  - Zero velocity model
  - Constant velocity model
  - Constant acceleration model
- The Kalman Filter
  - Kalman filter for 1D state
  - General Kalman filter
  - Limitations

Notation Reminder

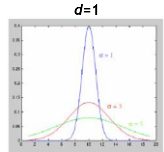
$$\mathbf{x} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

- Random variable with Gaussian probability distribution that has the mean vector μ and covariance matrix Σ.
- $\mathbf{x}$ and μ are d-dimensional, Σ is $d \times d$.

d=2



d=1



If $\mathbf{x}$ is 1D, we just have one Σ parameter: the variance σ^2

Linear Dynamic Models

- **Dynamics model**
  - State undergoes linear tranformation D_t plus Gaussian noise

$$\underset{n\times 1}{\mathbf{x}_t} \sim N\left(\underset{n\times n}{\mathbf{D}_t} \underset{n\times 1}{\mathbf{x}_{t-1}}, \Sigma_{d_t}\right)$$

- **Observation model**
  - Measurement is linearly transformed state plus Gaussian noise

$$\underset{m\times 1}{\mathbf{y}_t} \sim N\left(\underset{m\times n}{\mathbf{M}_t} \underset{n\times 1}{\mathbf{x}_t}, \Sigma_{m_t}\right)$$

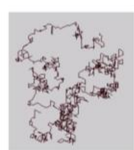
Example: Randomly Drifting Points

- Consider a stationary object, with state as position.
  - Position is constant, only motion due to random noise term.

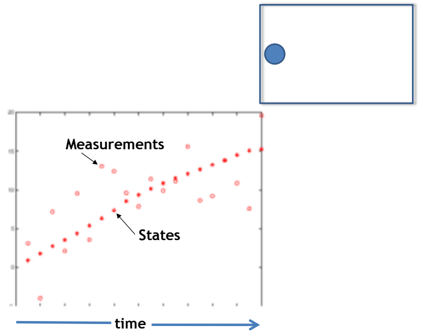
$$x_t = p_t \qquad p_t = p_{t-1} + \varepsilon$$

$\Rightarrow$ State evolution is described by identity matrix D=I

$$x_t = D_t x_{t-1} + noise = I p_{t-1} + noise$$



Example: Constant Velocity (1D Points)



Example: Constant Velocity (1D Points)

- State vector: position p and velocity v

$$x_t = \begin{bmatrix} p_t \\ v_t \end{bmatrix} \qquad p_t =$$

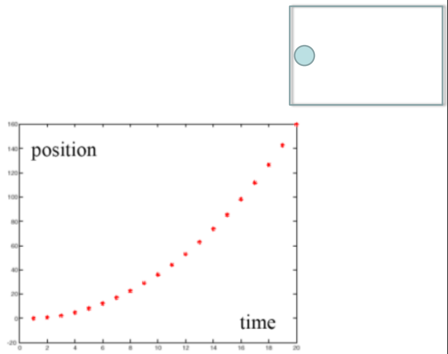
(greek letters denote noise terms)

$$x_t = D_t x_{t-1} + noise =$$

- Measurement is position only

$$y_t = Mx_t + noise =$$

Example: Constant Acceleration (1D Points)



Example: Constant Acceleration (1D Points)

- State vector: position p, velocity v, and acceleration a.

$$x_t = \begin{bmatrix} p_t \\ v_t \\ a_t \end{bmatrix} \qquad \begin{aligned} p_t &= p_{t-1} + (\Delta t) v_{t-1} + \varepsilon \\ v_t &= \\ a_t &= \end{aligned}$$

(greek letters denote noise terms)

$$x_t = D_t x_{t-1} + noise =$$

- Measurement is position only

$$y_t = Mx_t + noise =$$

Example: General Motion Models

- Assuming we have differential equations for the motion
  - E.g. for (undampened) periodic motion of a pendulum

$$\frac{d^2 p}{dt^2} = -p$$

- Substitute variables to transform this into linear system

$$p_1 = p \qquad p_2 = \frac{dp}{dt} \qquad p_3 = \frac{d^2 p}{dt^2}$$

- Then we have

$$x_t = \begin{bmatrix} p_{1,t} \\ p_{2,t} \\ p_{3,t} \end{bmatrix} \qquad \begin{aligned} p_{1,t} &= p_{1,t-1} + (\Delta t) p_{2,t-1} + \varepsilon \\ p_{2,t} &= p_{2,t-1} + (\Delta t) p_{3,t-1} + \xi \\ p_{3,t} &= -p_{1,t-1} + \zeta \end{aligned} \qquad D_t = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ -1 & 0 & 0 \end{bmatrix}$$

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  - Detection vs. Tracking
  - Tracking as probabilistic inference
  - Prediction and Correction
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  - Constant velocity model
  - Constant acceleration model
- **The Kalman Filter**
  - Kalman filter for 1D state
  - General Kalman filter
  - Limitations

The Kalman Filter

- Kalman filter
  - Method for tracking linear dynamical models in Gaussian noise
- The predicted/corrected state distributions are Gaussian
  - You only need to maintain the mean and covariance.
  - The calculations are easy (all the integrals can be done in closed form).

Slide credit: Svetlana Lazebnik

The Kalman Filter

Know corrected state from previous time step, and all measurements up to the current one
→ Predict distribution over next state.

Receive measurement

Know prediction of state, and next measurement
→ Update distribution over current state.

Time update ("Predict")

Measurement update ("Correct")

$P(X_t | y_0, \ldots, y_{t-1})$

$P(X_t | y_0, \ldots, y_t)$

Time advances: t++

Mean and std. dev. of predicted state:

μ_t^-, σ_t^-

Mean and std. dev. of corrected state:

μ_t^+, σ_t^+

Slide credit: Kristen Grauman

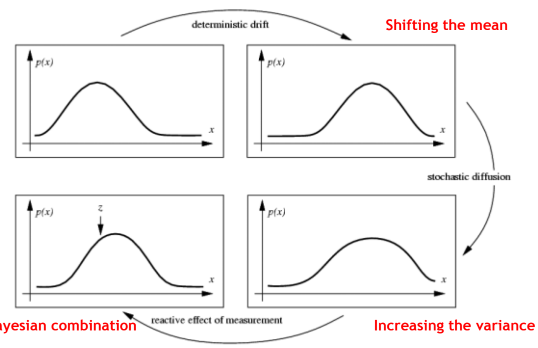
Kalman Filter for 1D State

Want to represent and update

$$P(x_t | y_0, \ldots, y_{t-1}) = N(\mu_t^-, (\sigma_t^-)^2)$$

$$P(x_t | y_0, \ldots, y_t) = N(\mu_t^+, (\sigma_t^+)^2)$$

Propagation of Gaussian densities



Slide credit: Svetlana Lazebnik

1D Kalman Filter: Prediction

- Have linear dynamic model defining predicted state evolution, with noise

$$X_t \sim N(dx_{t-1}, \sigma_d^2)$$

- Want to estimate predicted distribution for next state

$$P(X_t | y_0, \ldots, y_{t-1}) = N(\mu_t^-, (\sigma_t^-)^2)$$

- Update the mean:

$$\mu_t^- = d\mu_{t-1}^+$$

for derivations, see F&P Chapter 17.3

- Update the variance:

$$(\sigma_t^-)^2 = \sigma_d^2 + (d\sigma_{t-1}^+)^2$$

Slide credit: Kristen Grauman

1D Kalman Filter: Correction

- Have linear model defining the mapping of state to measurements: $Y_t \sim N(mx_t, \sigma_m^2)$
- Want to estimate corrected distribution given latest measurement: $P(X_t | y_0, \ldots, y_t) = N(\mu_t^+, (\sigma_t^+)^2)$
- Update the mean:

$$\mu_t^+ = \frac{\mu_t^- \sigma_m^2 + m y_t (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2}$$

- Update the variance:

$$(\sigma_t^+)^2 = \frac{\sigma_m^2 (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2}$$

Slide credit: Kristen Grauman

Derivations: F&P Chapter 17.3

Prediction vs. Correction

$$\mu_t^+ = \frac{\mu_t^- \sigma_m^2 + m y_t (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2} \qquad (\sigma_t^+)^2 = \frac{\sigma_m^2 (\sigma_t^-)^2}{\sigma_m^2 + m^2 (\sigma_t^-)^2}$$

- What if there is no prediction uncertainty $(\sigma_t^- = 0)$?

$$\mu_t^+ = \mu_t^- \qquad (\sigma_t^+)^2 = 0$$

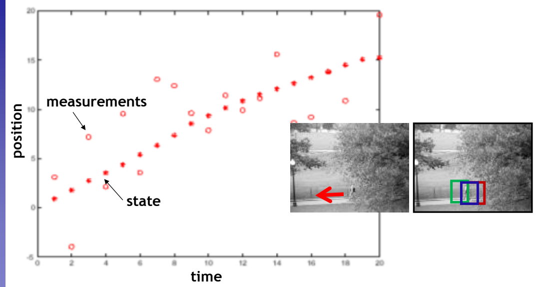
The measurement is ignored!

- What if there is no measurement uncertainty $(\sigma_m = 0)$?

$$\mu_t^+ = \frac{y_t}{m} \qquad (\sigma_t^+)^2 = 0$$

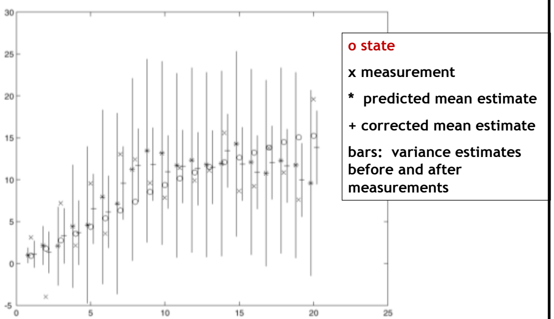
The prediction is ignored!

Recall: Constant Velocity Example

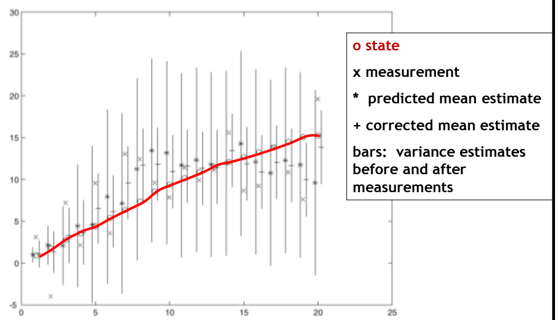


State is 2D: position + velocity
Measurement is 1D: position

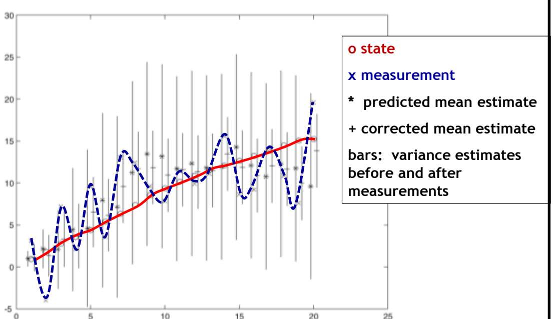
Constant Velocity Model



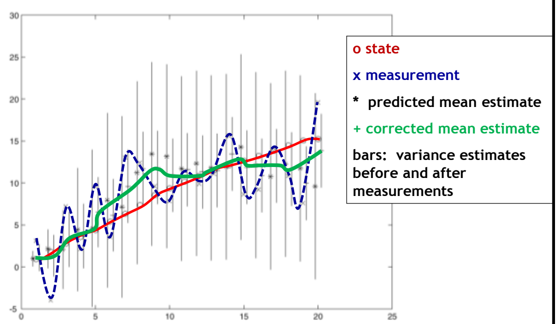
Constant Velocity Model



Constant Velocity Model



Constant Velocity Model



Kalman Filter: General Case (>1dim)

- What if state vectors have more than one dimension?

PREDICT

$$x_t^- = D_t x_{t-1}^+$$
$$\Sigma_t^- = D_t \Sigma_{t-1}^+ D_t^T + \Sigma_{d_t}$$

CORRECT

$$K_t = \Sigma_t^- M_t^T \left(M_t \Sigma_t^- M_t^T + \Sigma_{m_t}\right)^{-1}$$
$$x_t^+ = x_t^- + K_t\left(y_t - M_t x_t^-\right)$$ "residual"
$$\Sigma_t^+ = \left(I - K_t M_t\right)\Sigma_t^-$$

More weight on residual when measurement error covariance approaches 0.

Less weight on residual as a priori estimate error covariance approaches 0.

for derivations, see F&P Chapter 17.3

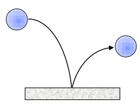
Summary: Kalman Filter

- **Pros:**
  - Gaussian densities everywhere
  - Simple updates, compact and efficient
  - Very established method, very well understood
- **Cons:**
  - Unimodal distribution, only single hypothesis
  - Restricted class of motions defined by linear model

Why Is This A Restriction?

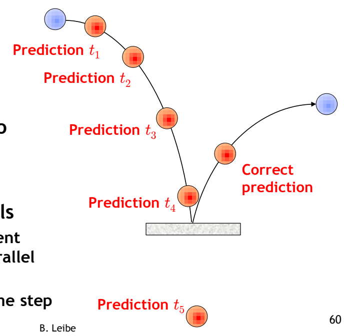
- Many interesting cases don't have linear dynamics
  - E.g. pedestrians walking

  - E.g. a ball bouncing



Ball Example: What Goes Wrong Here?

- Assuming constant acceleration model
- Prediction is too far from true position to compensate…
- Possible solution: Keep multiple models
  - Keep multiple different motion models in parallel
  - I.e. would check for bouncing at each time step



References and Further Reading

- A very good introduction to tracking with linear dynamic models and Kalman filters can be found in Chapter 17 of
  - D. Forsyth, J. Ponce, *Computer Vision - A Modern Approach*. Prentice Hall, 2003