Computer Vision II - Lecture 3

Template Tracking

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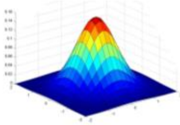
Course Outline

- Single-Object Tracking
  - Background modeling
  - Template based tracking
  - Color based tracking
  - Contour based tracking
  - Tracking by online classification
  - Tracking-by-detection
- Bayesian Filtering
- Multi-Object Tracking
- Articulated Tracking

Image source: Robert Collins

Recap: Gaussian Background Model

- Statistical model
  - Value of a pixel represents a measurement of the radiance of the first object intersected by the pixel's optical ray.
  - With a static background and static lighting, this value will be a constant affected by i.i.d. Gaussian noise.



- Idea
  - Model the background distribution of each pixel by a single Gaussian centered at the mean pixel value:

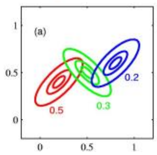
$$\mathcal{N}(\mathbf{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right\}$$

  - Test if a newly observed pixel value has a high likelihood under this Gaussian model.

$\Rightarrow$ Automatic estimation of a sensitivity threshold for each pixel.

MoG Background Model

- Improved statistical model
  - Large jumps between different pixel values because different objects are projected onto the same pixel at different times.
  - While the same object is projected onto the pixel, small local intensity variations due to Gaussian noise.



- Idea
  - Model the color distribution of each pixel by a mixture of K Gaussians

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

  - Evaluate likelihoods of observed pixel values under this model.
  - Or let entire Gaussian components adapt to foreground objects and classify components as belonging to object or background.

Image source: Chris Bischop

Recap: Stauffer-Grimson Background Model

- Idea
  - Model the distribution of each pixel by a mixture of K Gaussians

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k) \quad \text{where} \quad \boldsymbol{\Sigma}_k = \sigma_k^2 \mathbf{I}$$

  - Check every new pixel value against the existing K components until a match is found (pixel value within $2.5\ \sigma_k$ of μ_k).
  - If a match is found, adapt the corresponding component.
  - Else, replace the least probable component by a distribution with the new value as its mean and an initially high variance and low prior weight.
  - Order the components by the value of w_k/σ_k and select the best B components as the background model, where

$$B = \arg\min_b \left(\sum_{k=1}^{b} \frac{w_k}{\sigma_k} > T \right)$$

[C. Stauffer, W.E.L. Grimson, CVPR'99]

Recap: Stauffer-Grimson Background Model

- Online adaptation
  - Instead of estimating the MoG using EM, use a simpler online adaptation, assigning each new value only to the matching component.
  - Let $M_{k,t} = 1$ iff component k is the model that matched, else 0.

$$\pi_k^{(t+1)} = (1-\alpha)\pi_k^{(t)} + \alpha M_{k,t}$$

  - Adapt only the parameters for the matching component

$$\boldsymbol{\mu}_k^{(t+1)} = (1-\rho)\boldsymbol{\mu}_k^{(t)} + \rho x^{(t+1)}$$

$$\boldsymbol{\Sigma}_k^{(t+1)} = (1-\rho)\boldsymbol{\Sigma}_k^{(t)} + \rho(x^{(t+1)} - \boldsymbol{\mu}_k^{(t+1)})(x^{(t+1)} - \boldsymbol{\mu}_k^{(t+1)})^T$$

where

$$\rho = \alpha \mathcal{N}(\mathbf{x}_n|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$$

(i.e., the update is weighted by the component likelihood)

[C. Stauffer, W.E.L. Grimson, CVPR'99]

Recap: Kernel Background Modeling

- Nonparametric density estimation
  - Estimate a pixel's background distribution using the kernel density estimator $K(\cdot)$ as

$$p(\mathbf{x}^{(t)}) = \frac{1}{N}\sum_{i=1}^{N} K(\mathbf{x}^{(t)} - \mathbf{x}^{(i)})$$

  - Choose K to be a Gaussian $\mathcal{N}(0, \mathbf{\Sigma})$ with $\mathbf{\Sigma} = \mathrm{diag}\{\sigma_j\}$. Then

$$p(\mathbf{x}^{(t)}) = \frac{1}{N}\sum_{i=1}^{N}\prod_{j=1}^{d}\frac{1}{\sqrt{2\pi\sigma_j^2}}e^{-\frac{1}{2}\frac{(x_j^{(t)}-x_j^{(i)})^2}{\sigma_j^2}}$$

  - A pixel is considered foreground if $p(\mathbf{x}^{(t)}) < \theta$ for a threshold θ.
    - This can be computed very fast using lookup tables for the kernel function values, since all inputs are discrete values.
    - Additional speedup: partial evaluation of the sum usually sufficient

[A. Elgammal, D. Harwood, L. Davis, ECCV'00]

Applications: Visual Surveillance

- Background modeling to detect objects for tracking
  - Extension: Learning a foreground model for each object.

Video source: Ian Reid, Univ. of Oxford

Applications: Articulated Tracking

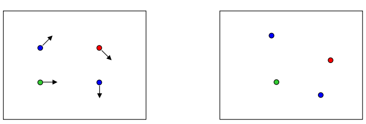
- Background modeling as preprocessing step
  - Track a person's location through the scene
  - Extract silhouette information from the foreground mask.
  - Perform body pose estimation based on this mask.

Video source: Hedvik Kjellstroem, Tobias Jaeggli

Topics of This Lecture

- Recap: Lucas-Kanade Optical Flow
  - Brightness Constancy constraint
  - LK flow estimation
  - Coarse-to-fine estimation
- Feature Tracking
  - KLT feature tracking
- Template Tracking
  - LK derivation for templates
  - Warping functions
  - General LK image registration
- Applications

Recap: Estimating Optical Flow

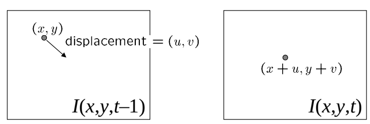


$I(x,y,t-1)$ $I(x,y,t)$

- Optical Flow
  - Given two subsequent frames, estimate the apparent motion field $u(x,y)$ and $v(x,y)$ between them.
- Key assumptions
  - Brightness constancy: projection of the same point looks the same in every frame.
  - Small motion: points do not move very far.
  - Spatial coherence: points move like their neighbors.

Slide credit: Svetlana Lazebnik

Recap: The Brightness Constancy Constraint



- Brightness Constancy Equation:

$$I(x,y,t-1) = I(x+u(x,y), y+v(x,y), t)$$

- Linearizing the right hand side using Taylor expansion:

$$I(x,y,t-1) \approx I(x,y,t) + I_x \cdot u(x,y) + I_y \cdot v(x,y)$$

- Hence, $I_x \cdot u + I_y \cdot v + I_t \approx 0$

Spatial derivatives — Temporal derivative

Slide credit: Svetlana Lazebnik

Recap: The Brightness Constancy Constraint

$$I_x \cdot u + I_y \cdot v + I_t = 0$$

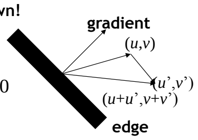
- How many equations and unknowns per pixel?
  - One equation, two unknowns
- Intuitively, what does this constraint mean?

$$\nabla I \cdot (u, v) + I_t = 0$$

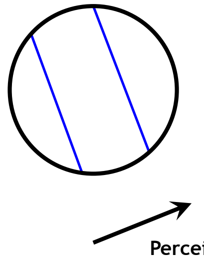
  - It gives us a constraint on the component of the flow in the direction of the gradient.

$\Rightarrow$ The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown!

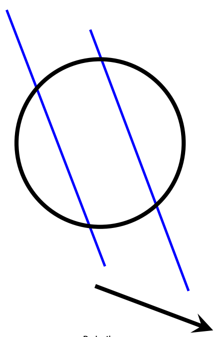
If (u,v) satisfies the equation, so does $(u+u', v+v')$ if $\nabla I \cdot (u', v') = 0$



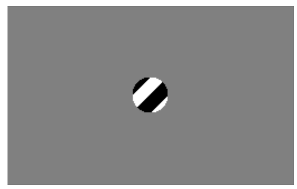
The Aperture Problem



The Aperture Problem



The Barber Pole Illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

The Barber Pole Illusion

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Recap: Solving the Aperture Problem

- How to get more equations for a pixel?
- Spatial coherence constraint
  - Pretend the pixel's neighbors have the same (u,v).
  - If we use a 5×5 window, that gives us 25 equations per pixel

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \; v]$$

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}$$

B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proc. IJCAI'81*, pp. 674-679, 1981.

Recap: Solving the Aperture Problem

- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix} \qquad \underset{25\text{x}2}{A} \; \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b}$$

- Minimum least squares solution given by solution of

$$\underset{2\text{x}2}{(A^T A)} \; \underset{2\text{x}1}{d} = \underset{2\text{x}1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

(The summations are over all pixels in the $K \times K$ window)

Recap: Conditions for Solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- When is this solvable?
  - $A^T A$ should be invertible.
  - $A^T A$ entries should not be too small (noise).
  - $A^T A$ should be well-conditioned.

  ⇒ Looking for cases where A has two large eigenvalues (i.e., corners and highly textured areas).

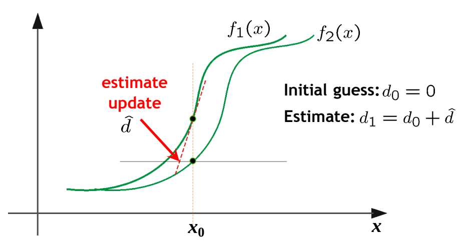
Recap: Iterative LK Refinement

1. Estimate velocity at each pixel using one iteration of LK estimation.

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

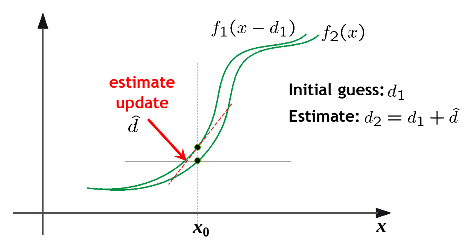
2. Warp one image toward the other using the estimated flow field.
   - *(Easier said than done)*
3. Refine estimate by repeating the process.

Recap: Iterative LK Refinement



(using d for *displacement* here instead of u)

Recap: Iterative LK Refinement



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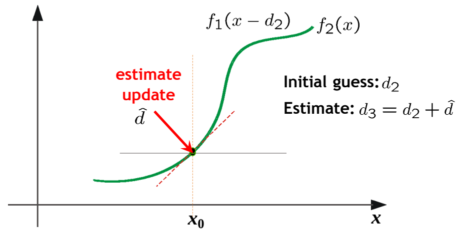
Recap: Iterative LK Refinement



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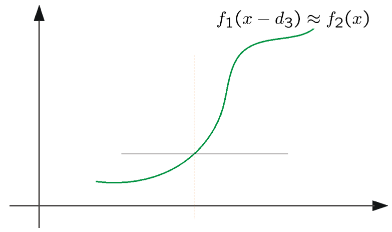
Recap: Iterative LK Refinement

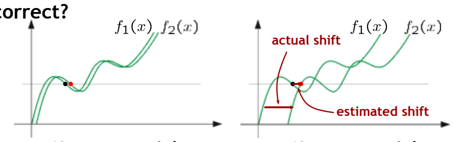


(using d for *displacement* here instead of u)

Problem Case: Large Motions

Temporal Aliasing

- Temporal aliasing causes ambiguities in optical flow because images can have many pixels with the same intensity.
- I.e., how do we know which 'correspondence' is correct?

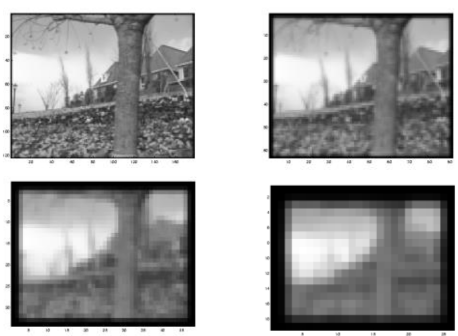


Nearest match is correct (no aliasing)

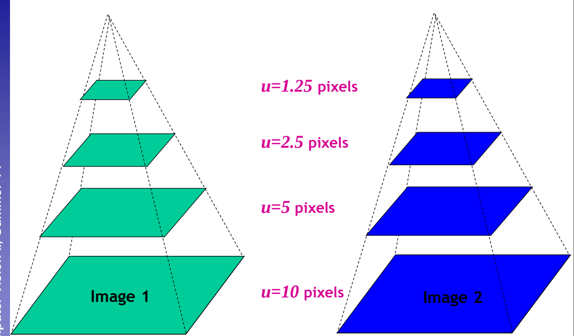
Nearest match is incorrect (aliasing)

- To overcome aliasing: coarse-to-fine estimation.

Idea: Reduce the Resolution!



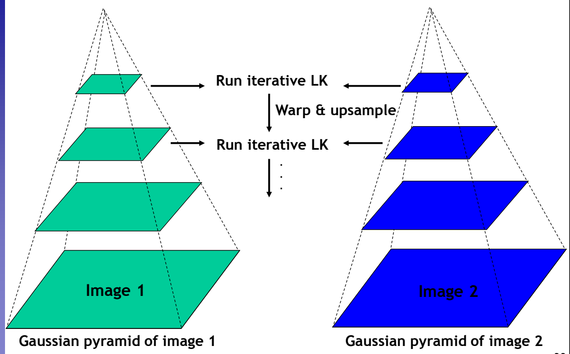
Recap: Coarse-to-fine Optical Flow Estimation



Gaussian pyramid of image 1

Gaussian pyramid of image 2

Recap: Coarse-to-fine Optical Flow Estimation



Slide credit: Steve Seitz

Topics of This Lecture

- Recap: Lucas-Kanade Optical Flow
  - Brightness Constancy constraint
  - LK flow estimation
  - Coarse-to-fine estimation
- **Feature Tracking**
  - KLT feature tracking
- Template Tracking
  - LK derivation for templates
  - Warping functions
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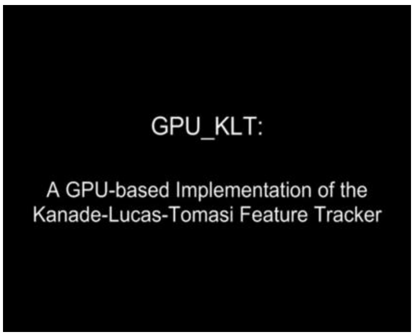
- Applications

KLT Feature Tracking



http://www.cs.unc.edu/~ssinha/Research/GPU_KLT/

Shi-Tomasi Feature Tracker

- Idea
  - Find good features using eigenvalues of second-moment matrix
  - Key idea: "good" features to track are the ones that can be tracked reliably.
- Frame-to-frame tracking
  - Track with LK and a pure *translation* motion model.
  - More robust for small displacements, can be estimated from smaller neighborhoods (e.g., 5×5 pixels).
- Checking consistency of tracks
  - *Affine* registration to the first observed feature instance.
  - Affine model is more accurate for larger displacements.
  - Comparing to the first frame helps to minimize drift.

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

Slide credit: Svetlana Lazebnik

Tracking Example

Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.

Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.

Slide credit: Svetlana Lazebnik

Real-Time GPU Implementations

- This basic feature tracking framework (Lucas-Kanade + Shi-Tomasi) is commonly referred to as "KLT tracking".
  - Used as preprocessing step for many applications
  - Lends itself to easy parallelization
- Very fast GPU implementations available, e.g.,
  - C. Zach, D. Gallup, J.-M. Frahm, Fast Gain-Adaptive KLT tracking on the GPU. In CVGPU'08 Workshop, Anchorage, USA, 2008
  - 216 fps with automatic gain adaptation
  - 260 fps without gain adaptation

http://www.cs.unc.edu/~ssinha/Research/GPU_KLT/

http://www.inf.ethz.ch/personal/chzach/opensource.html

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Lucas-Kanade Template Tracking

- Traditional LK
  - Typically run on small, corner-like features (e.g., 5×5 patches) to compute optical flow (→ KLT).
  - However, there is no reason why we can't use the same approach on a larger window around the tracked object.

Basic LK Derivation for Templates

$$E(u,v) = \sum_{\mathbf{x}} \left[I(x+u, y+v) - T(x,y) \right]^2$$


Current frame


Template model

(u,v) = hypothesized location of template in current frame

Basic LK Derivation for Templates

- Taylor expansion

$$\begin{aligned} E(u,v) &= \sum_{\mathbf{x}} \left[I(x+u, y+v) - T(x,y) \right]^2 \\ &\approx \sum_{\mathbf{x}} \left[I(x,y) + uI_x(x,y) + vI_y(x,y) - T(x,y) \right]^2 \\ &= \sum_{\mathbf{x}} \left[uI_x(x,y) + vI_y(x,y) + D(x,y) \right]^2 \end{aligned}$$ with $D = I - T$

- Taking partial derivatives

$$\frac{\partial E}{\partial u} = \sum_{\mathbf{x}} \left[uI_x(x,y) + vI_y(x,y) + D(x,y) \right] I_x(x,y) \stackrel{!}{=} 0$$

$$\frac{\partial E}{\partial v} = \sum_{\mathbf{x}} \left[uI_x(x,y) + vI_y(x,y) + D(x,y) \right] I_y(x,y) \stackrel{!}{=} 0$$

- Equation in matrix form

$$\sum_{\mathbf{x}} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \sum_{\mathbf{x}} \begin{bmatrix} I_x D \\ I_y D \end{bmatrix}$$ ⇒ Solve via least-squares

One Problem With This...

- Problematic Assumption
  - Assumption of constant flow (pure translation) for all pixels in a larger window is unreasonable for long periods of time.

- However...
  - We can easily generalize the LK approach to other 2D parametric motion models (like affine or projective) by introducing a "warp" function $\mathbf{W}$ with parameters $\mathbf{p}$.

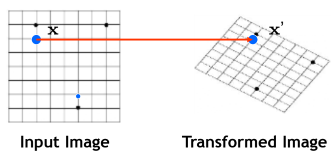
$$E(u,v) = \sum_{\mathbf{x}} \left[I(x+u, y+v) - T(x,y) \right]^2$$

$$\downarrow$$

$$E(\mathbf{p}) = \sum_{\mathbf{x}} \left[I(\mathbf{W}([x,y];\mathbf{p})) - T([x,y]) \right]^2$$

Geometric Image Warping

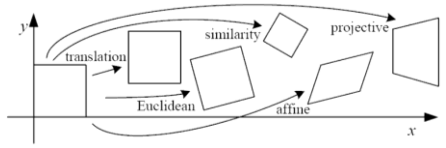
- The warp $\mathbf{W}(\mathbf{x};\mathbf{p})$ describes the geometric relationship between two images



$$\mathbf{x}' = \mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix} W_x(\mathbf{x};\mathbf{p}) \\ W_y(\mathbf{x};\mathbf{p}) \end{bmatrix}$$

Parameters of the warp

Example Warping Functions



Translation | Affine | Perspective | 3D rotation

2 unknowns | 6 unknowns | 8 unknowns | 3 unknowns

Example Warping Functions

- Translation

$$\mathbf{W}([x,y];\mathbf{p}) = \begin{bmatrix} x+p_1 \\ y+p_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & p_1 \\ 0 & 1 & p_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Affine

$$\mathbf{W}([x,y];\mathbf{p}) = \begin{bmatrix} x+p_1x+p_3y+p_5 \\ y+p_2x+p_4y+p_6 \end{bmatrix} = \begin{bmatrix} 1+p_1 & p_3 & p_5 \\ p_2 & 1+p_4 & p_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Perspective

$$\mathbf{W}([x,y];\mathbf{p}) = \frac{1}{p_7x+p_8y+1} \begin{bmatrix} x+p_1x+p_3y+p_5 \\ y+p_2x+p_4y+p_6 \end{bmatrix}$$

  - Note: Other parametrizations are possible; the above ones are just particularly convenient here.

General LK Image Registration

- Goal
  - Find the warping parameters $\mathbf{p}$ that minimize the sum-of-squares intensity difference between the template image and the warped input image.
- LK formulation
  - Formulate this as an optimization problem

$$\arg\min_{\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) - T(\mathbf{x}) \right]^2$$

  - We assume that an initial estimate of $\mathbf{p}$ is known and iteratively solve for increments to the parameters $\Delta\mathbf{p}$:

$$\arg\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x}) \right]^2$$

Step-by-Step Derivation

- Key to the derivation
  - Taylor expansion around $\Delta\mathbf{p}$

$$I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) \approx I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} + \mathcal{O}(\Delta\mathbf{p}^2)$$

  - Using pixel coordinates $\mathbf{x} = [x,y]$

$$\begin{aligned} I(\mathbf{W}([x,y];\mathbf{p}+\Delta\mathbf{p})) \approx\ & I(\mathbf{W}([x,y];p_1,\ldots,p_n)) \\ & + \left[\frac{\partial I}{\partial x}\frac{\partial W_x}{\partial p_1} + \frac{\partial I}{\partial y}\frac{\partial W_y}{\partial p_1} \right]_{p_1} \Delta p_1 \\ & + \left[\frac{\partial I}{\partial x}\frac{\partial W_x}{\partial p_2} + \frac{\partial I}{\partial y}\frac{\partial W_y}{\partial p_2} \right]_{p_1} \Delta p_2 \\ & + \ \ldots \\ & + \left[\frac{\partial I}{\partial x}\frac{\partial W_x}{\partial p_n} + \frac{\partial I}{\partial y}\frac{\partial W_y}{\partial p_n} \right]_{p_n} \Delta p_n \end{aligned}$$

Step-by-Step Derivation

- Rewriting this in matrix notation

$$\begin{aligned} I(\mathbf{W}([x,y];\mathbf{p}+\Delta\mathbf{p})) \approx\ & I(\mathbf{W}([x,y];p_1,\ldots,p_n)) \\ & + \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_1} \\ \frac{\partial W_y}{\partial p_1} \end{bmatrix}_{p_1} \Delta p_1 \\ & + \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_2} \\ \frac{\partial W_y}{\partial p_2} \end{bmatrix}_{p_2} \Delta p_2 \\ & + \ \ldots \\ & + \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_n} \\ \frac{\partial W_y}{\partial p_n} \end{bmatrix}_{p_n} \Delta p_n \end{aligned}$$

Step-by-Step Derivation

- And further collecting the derivative terms

$$\begin{aligned} I(\mathbf{W}([x,y];\mathbf{p}+\Delta\mathbf{p})) \approx\ & I(\mathbf{W}([x,y];p_1,\ldots,p_n)) \\ & + \begin{bmatrix} \frac{\partial I}{\partial x} & \frac{\partial I}{\partial y} \end{bmatrix} \begin{bmatrix} \frac{\partial W_x}{\partial p_1} & \frac{\partial W_x}{\partial p_2} & \cdots & \frac{\partial W_x}{\partial p_n} \\ \frac{\partial W_y}{\partial p_1} & \frac{\partial W_y}{\partial p_2} & \cdots & \frac{\partial W_y}{\partial p_n} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_n \end{bmatrix} \end{aligned}$$

Gradient: ∇I

Jacobian: $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$

Increment parameters to solve for: $\Delta\mathbf{p}$

  - Written in matrix form

$$I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) \approx I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p}$$

Example: Jacobian of Affine Warp

- General equation of Jacobian

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{bmatrix} \frac{\partial W_x}{\partial p_1} & \frac{\partial W_x}{\partial p_2} & \cdots & \frac{\partial W_x}{\partial p_n} \\ \frac{\partial W_y}{\partial p_1} & \frac{\partial W_y}{\partial p_2} & \cdots & \frac{\partial W_y}{\partial p_n} \end{bmatrix}$$

- Affine warp function (6 parameters)

$$\mathbf{W}([x,y];\mathbf{p}) = \begin{bmatrix} 1+p_1 & p_3 & p_5 \\ p_2 & 1+p_4 & p_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Result

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \frac{\partial \begin{bmatrix} x+p_1x+p_3y+p_5 \\ p_2x+y+p_4y+p_6 \end{bmatrix}}{\partial \mathbf{p}}$$

$$= \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$$

Slide credit: Robert Collins

Minimizing the Registration Error

- Optimization function after Taylor expansion

$$\arg\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]^2$$

- Minimizing this function
  - *How?*

Minimizing the Registration Error

- Optimization function after Taylor expansion

$$\arg\min_{\Delta\mathbf{p}} \sum_{\mathbf{x}} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]^2$$

- Minimizing this function
  - Taking the partial derivative and setting it to zero

$$\frac{\partial}{\partial \Delta\mathbf{p}} \stackrel{!}{=} 0 \quad \rangle \; 2\sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right] \stackrel{!}{=} 0$$

  - Closed-form solution for $\Delta\mathbf{p}$ (Gauss-Newton):

$$\Delta\mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p})) \right]$$

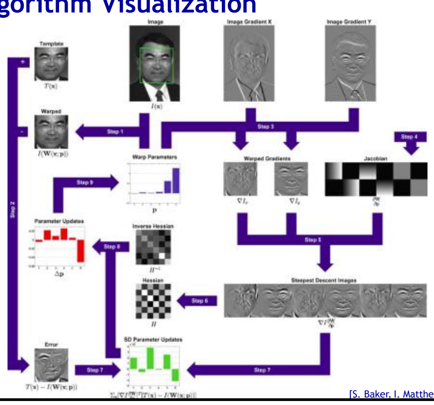
  - where $\mathbf{H}$ is the Hessian $\mathbf{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$

Summary: LK Algorithm

- Iterate
  - Warp I to obtain $I(\mathbf{W}([x,y];\mathbf{p}))$
  - Compute the error image $T([x,y]) - I(\mathbf{W}([x,y];\mathbf{p}))$
  - Warp the gradient ∇I with $\mathbf{W}([x,y];\mathbf{p})$
  - Evaluate $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at $([x,y];\mathbf{p})$ (Jacobian)
  - Compute steepest descent images $\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$
  - Compute Hessian matrix $\mathbf{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$
  - Compute $\sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[T([x,y]) - I(\mathbf{W}([x,y];\mathbf{p})) \right]$
  - Compute $\Delta\mathbf{p} = \mathbf{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^T \left[T([x,y]) - I(\mathbf{W}([x,y];\mathbf{p})) \right]$
  - Update the parameters $\mathbf{p} \leftarrow \mathbf{p} + \Delta\mathbf{p}$
- Until $\Delta\mathbf{p}$ magnitude is negligible

[S. Baker, I. Matthews, IJCV'04]

LK Algorithm Visualization



[S. Baker, I. Matthews, IJCV'04]

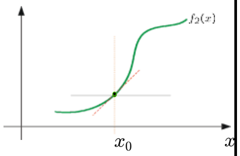
Discussion LK Alignment

- Pros
  - All pixels get used in matching
  - Can get sub-pixel accuracy (important for good mosaicking)
  - Fast and simple algorithm
  - Applicable to Optical Flow estimation, stereo disparity estimation, parametric motion tracking, etc.
- Cons
  - Prone to local minima.
  - Relatively small movement.
  - ⇒ Good initialization necessary

Slide credit: Robert Collins

Side Note

- LK Registration needs a good initialization
  - Taylor expansion corresponds to a linearization around the initial position $\mathbf{p}$.
  - This linearization is only valid in a small neighborhood around $\mathbf{p}$.



- When tracking templates…
  - We typically use the previous frame's result as initialization.
  - ⇒ The higher the frame rate, the smaller the warp will be.
  - ⇒ This means we get better results and need fewer LK iterations.
  - ⇒ *Tracking becomes easier (and faster!) with higher frame rates.*

Discussion

- Beyond 2D Tracking/Registration
  - So far, we focused on registration between 2D images.
  - The same ideas can be used when performing registration between a 3D model and the 2D image (model-based tracking).
  - The approach can also be extended for dealing with articulated objects and for tracking in subspaces.

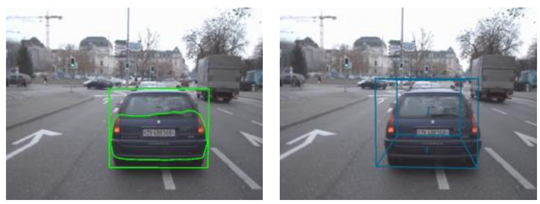
⇒ We will come back to this in later lectures when we talk about model-based 3D tracking…

Slide adapted from Jinxiang Chai

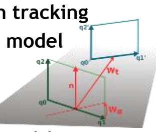
Topics of This Lecture

- Recap: Lucas-Kanade Optical Flow
  - Brightness Constancy constraint
  - LK flow estimation
  - Coarse-to-fine estimation
- Feature Tracking
  - KLT feature tracking
- Template Tracking
  - LK derivation for templates
  - Warping functions
  - General LK image registration
- **Applications**

Example of a More Complex Warping Function



- Encode geometric constraints into region tracking
- Constrained homography transformation model
  - Translation parallel to the ground plane
  - Rotation around the ground plane normal
  - $\mathbf{W}(\mathbf{x}) = \mathbf{W}_{obj}\ \mathbf{P}\ \mathbf{W}_t\ \mathbf{W}_\alpha\ \mathbf{Q}\ \mathbf{x}$



⇒ Input for high-level tracker with car steering model.

[E. Horbert, D. Mitzel, B. Leibe, DAGM'10]

References and Further Reading

- The original paper by Lucas & Kanade
  - B. Lucas and T. Kanade. An iterative image registration technique with an application to stereo vision. In *Proc. IJCAI*, pp. 674-679, 1981.
- A more recent paper giving a better explanation
  - S. Baker, I. Matthews. Lucas-Kanade 20 Years On: A Unifying Framework. In IJCV, Vol. 56(3), pp. 221-255, 2004.
- The original KLT paper by Shi & Tomasi
  - J. Shi and C. Tomasi. Good Features to Track. CVPR 1994.